تمارين محلولة: التحويل الخطية وحساب التكامل

تمرين 1

حساب نهاية دالة بعدة متغيرات

$\lim_{(x,y)\to(0,0)} \frac{x^2+y^2}{x+y+1}$ $\quad$ $\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x+y}$ $\quad$ $\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x-y}$

$\lim_{(x,y)\to(0,0)} \frac{x^2+y^2}{x+y+1}$ $\quad$ $\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x+y}$ $\quad$ $\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x-y}$

الحل

$$\lim_{(x,y)\to(0,0)} \frac{x^2+y^2}{x+y+1} = \frac{0+0}{0+0+1} = 0$$

$$\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x+y} = \lim_{(x,y)\to(0,0)} \frac{(x-y)(x+y)}{x+y} = \lim_{(x,y)\to(0,0)} (x-y) = 0-0 = 0$$

$$\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x-y} = \lim_{(x,y)\to(0,0)} \frac{(x-y)(x+y)}{x-y} = \lim_{(x,y)\to(0,0)} (x+y) = 0+0 = 0$$

تمرين 2

حساب نهاية دالة بعدة متغيرات

$\lim_{(x,y)\to(0,0)} \frac{x^2+y^2}{x+y}$ $\quad$ $\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x-y}$

الحل

$$\lim_{(x,y)\to(0,0)} \frac{x^2+y^2}{x+y} = \lim_{(x,y)\to(0,0)} \frac{(x+y)^2-2xy}{x+y} = \lim_{(x,y)\to(0,0)} (x+y) - \lim_{(x,y)\to(0,0)} \frac{2xy}{x+y}$$

$$= 0+0 - \lim_{(x,y)\to(0,0)} \frac{2xy}{x+y}$$

$$\lim_{(x,y)\to(0,0)} \frac{2xy}{x+y} = \lim_{(x,y)\to(0,0)} \frac{2x^2+2xy}{x+y} = \lim_{(x,y)\to(0,0)} \frac{2x(x+y)}{x+y} = \lim_{(x,y)\to(0,0)} 2x = 2\times 0 = 0$$

$$\lim_{(x,y)\to(0,0)} \frac{x^2+y^2}{x+y} = 0 - 0 = 0$$

$$\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x-y} = \lim_{(x,y)\to(0,0)} \frac{(x-y)(x+y)}{x-y} = \lim_{(x,y)\to(0,0)} (x+y) = 0+0 = 0$$

تمرين 3

حساب نهاية دالة بعدة متغيرات

$\lim_{(x,y)\to(0,0)} \frac{x^2+y^2}{x+y}$ $\quad$ $\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x-y}$

الحل

$$\lim_{(x,y)\to(0,0)} \frac{x^2+y^2}{x+y} = \lim_{(x,y)\to(0,0)} \frac{(x+y)^2-2xy}{x+y} = \lim_{(x,y)\to(0,0)} (x+y) - \lim_{(x,y)\to(0,0)} \frac{2xy}{x+y}$$

$$= 0+0 - \lim_{(x,y)\to(0,0)} \frac{2xy}{x+y}$$

$$\lim_{(x,y)\to(0,0)} \frac{2xy}{x+y} = \lim_{(x,y)\to(0,0)} \frac{2x^2+2xy}{x+y} = \lim_{(x,y)\to(0,0)} \frac{2x(x+y)}{x+y} = \lim_{(x,y)\to(0,0)} 2x = 2\times 0 = 0$$

$$\lim_{(x,y)\to(0,0)} \frac{x^2+y^2}{x+y} = 0 - 0 = 0$$

$$\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x-y} = \lim_{(x,y)\to(0,0)} \frac{(x-y)(x+y)}{x-y} = \lim_{(x,y)\to(0,0)} (x+y) = 0+0 = 0$$

تمرين 4

حساب نهاية دالة بعدة متغيرات

$\lim_{(x,y)\to(0,0)} \frac{x^2}{x+y}$ $\quad$ $\lim_{(x,y)\to(0,0)} \frac{x^2}{x-y}$

الحل

$$\lim_{(x,y)\to(0,0)} \frac{x^2}{x+y} = \lim_{(x,y)\to(0,0)} \frac{x^2+xy-xy}{x+y}$$

$$\lim_{(x,y)\to(0,0)} \frac{x^2+xy}{x+y} = \lim_{(x,y)\to(0,0)} \frac{x(x+y)}{x+y} = \lim_{(x,y)\to(0,0)} x = 0$$

$$\lim_{(x,y)\to(0,0)} \frac{-xy}{x+y} = \lim_{(x,y)\to(0,0)} \frac{-xy}{x} = \lim_{(x,y)\to(0,0)} -y = -0 = 0$$

$$\lim_{(x,y)\to(0,0)} \frac{x^2}{x+y} = 0 + 0 = 0$$

$$\lim_{(x,y)\to(0,0)} \frac{x^2}{x-y} = \lim_{(x,y)\to(0,0)} \frac{x^2-xy+xy}{x-y}$$

$$\lim_{(x,y)\to(0,0)} \frac{x^2-xy}{x-y} = \lim_{(x,y)\to(0,0)} \frac{x(x-y)}{x-y} = \lim_{(x,y)\to(0,0)} x = 0$$

$$\lim_{(x,y)\to(0,0)} \frac{xy}{x-y} = \lim_{(x,y)\to(0,0)} \frac{xy}{x} = \lim_{(x,y)\to(0,0)} y = 0$$

$$\lim_{(x,y)\to(0,0)} \frac{x^2}{x-y} = 0 + 0 = 0$$

تمرين 5

حساب نهاية دالة بعدة متغيرات

$\lim_{(x,y)\to(0,0)} \frac{x^2+y^2}{x+y+1}$ $\quad$ $\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x+y}$

$\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x-y}$

الحل

$$\lim_{(x,y)\to(0,0)} \frac{x^2+y^2}{x+y+1} = \frac{0+0}{0+0+1} = 0$$

$$\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x+y} = \lim_{(x,y)\to(0,0)} \frac{(x-y)(x+y)}{x+y} = \lim_{(x,y)\to(0,0)} (x-y) = 0-0 = 0$$

$$\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x-y} = \lim_{(x,y)\to(0,0)} \frac{(x-y)(x+y)}{x-y} = \lim_{(x,y)\to(0,0)} (x+y) = 0+0 = 0$$

تمرين 6

حساب نهاية دالة بعدة متغيرات

$\lim_{(x,y)\to(0,0)} \frac{x^2+y^2}{x+y}$ $\quad$ $\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x-y}$

الحل

$$\lim_{(x,y)\to(0,0)} \frac{x^2+y^2}{x+y} = \lim_{(x,y)\to(0,0)} \frac{(x+y)^2-2xy}{x+y} = \lim_{(x,y)\to(0,0)} (x+y) - \lim_{(x,y)\to(0,0)} \frac{2xy}{x+y}$$

$$= 0+0 - \lim_{(x,y)\to(0,0)} \frac{2xy}{x+y}$$

$$\lim_{(x,y)\to(0,0)} \frac{2xy}{x+y} = \lim_{(x,y)\to(0,0)} \frac{2x^2+2xy}{x+y} = \lim_{(x,y)\to(0,0)} \frac{2x(x+y)}{x+y} = \lim_{(x,y)\to(0,0)} 2x = 2\times 0 = 0$$

$$\lim_{(x,y)\to(0,0)} \frac{x^2+y^2}{x+y} = 0 - 0 = 0$$

$$\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x-y} = \lim_{(x,y)\to(0,0)} \frac{(x-y)(x+y)}{x-y} = \lim_{(x,y)\to(0,0)} (x+y) = 0+0 = 0$$

تمرين 7

حساب نهاية دالة بعدة متغيرات

$\lim_{(x,y)\to(0,0)} \frac{x^2+y^2}{x+y}$ $\quad$ $\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x-y}$

الحل

$$\lim_{(x,y)\to(0,0)} \frac{x^2+y^2}{x+y} = \lim_{(x,y)\to(0,0)} \frac{(x+y)^2-2xy}{x+y} = \lim_{(x,y)\to(0,0)} (x+y) - \lim_{(x,y)\to(0,0)} \frac{2xy}{x+y}$$

$$= 0+0 - \lim_{(x,y)\to(0,0)} \frac{2xy}{x+y}$$

$$\lim_{(x,y)\to(0,0)} \frac{2xy}{x+y} = \lim_{(x,y)\to(0,0)} \frac{2x^2+2xy}{x+y} = \lim_{(x,y)\to(0,0)} \frac{2x(x+y)}{x+y} = \lim_{(x,y)\to(0,0)} 2x = 2\times 0 = 0$$

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$$\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x-y} = \lim_{(x,y)\to(0,0)} \frac{(x-y)(x+y)}{x-y} = \lim_{(x,y)\to(0,0)} (x+y) = 0+0 = 0$$

تمرين 8

حساب نهاية دالة بعدة متغيرات

$\lim_{(x,y)\to(0,0)} \frac{x^2+y^2}{x+y}$ $\quad$ $\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x-y}$

الحل

$$\lim_{(x,y)\to(0,0)} \frac{x^2+y^2}{x+y} = \lim_{(x,y)\to(0,0)} \frac{(x+y)^2-2xy}{x+y} = \lim_{(x,y)\to(0,0)} (x+y) - \lim_{(x,y)\to(0,0)} \frac{2xy}{x+y}$$

$$= 0+0 - \lim_{(x,y)\to(0,0)} \frac{2xy}{x+y}$$

$$\lim_{(x,y)\to(0,0)} \frac{2xy}{x+y} = \lim_{(x,y)\to(0,0)} \frac{2x^2+2xy}{x+y} = \lim_{(x,y)\to(0,0)} \frac{2x(x+y)}{x+y} = \lim_{(x,y)\to(0,0)} 2x = 2\times 0 = 0$$

$$\lim_{(x,y)\to(0,0)} \frac{x^2+y^2}{x+y} = 0 - 0 = 0$$

$$\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x-y} = \lim_{(x,y)\to(0,0)} \frac{(x-y)(x+y)}{x-y} = \lim_{(x,y)\to(0,0)} (x+y) = 0+0 = 0$$

تمرين 9

حساب نهاية دالة بعدة متغيرات

$\lim_{(x,y)\to(0,0)} \frac{x^2+y^2}{x+y}$ $\quad$ $\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x-y}$

الحل

$$\lim_{(x,y)\to(0,0)} \frac{x^2+y^2}{x+y} = \lim_{(x,y)\to(0,0)} \frac{(x+y)^2-2xy}{x+y} = \lim_{(x,y)\to(0,0)} (x+y) - \lim_{(x,y)\to(0,0)} \frac{2xy}{x+y}$$

$$= 0+0 - \lim_{(x,y)\to(0,0)} \frac{2xy}{x+y}$$

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$$\lim_{(x,y)\to(0,0)} \frac{x^2+y^2}{x+y} = 0 - 0 = 0$$

$$\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x-y} = \lim_{(x,y)\to(0,0)} \frac{(x-y)(x+y)}{x-y} = \lim_{(x,y)\to(0,0)} (x+y) = 0+0 = 0$$

تمرين 10

حساب نهاية دالة بعدة متغيرات

$\lim_{(x,y)\to(0,0)} \frac{x^2+y^2}{x+y}$ $\quad$ $\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x-y}$

الحل

$$\lim_{(x,y)\to(0,0)} \frac{x^2+y^2}{x+y} = \lim_{(x,y)\to(0,0)} \frac{(x+y)^2-2xy}{x+y} = \lim_{(x,y)\to(0,0)} (x+y) - \lim_{(x,y)\to(0,0)} \frac{2xy}{x+y}$$

$$= 0+0 - \lim_{(x,y)\to(0,0)} \frac{2xy}{x+y}$$

$$\lim_{(x,y)\to(0,0)} \frac{2xy}{x+y} = \lim_{(x,y)\to(0,0)} \frac{2x^2+2xy}{x+y} = \lim_{(x,y)\to(0,0)} \frac{2x(x+y)}{x+y} = \lim_{(x,y)\to(0,0)} 2x = 2\times 0 = 0$$

$$\lim_{(x,y)\to(0,0)} \frac{x^2+y^2}{x+y} = 0 - 0 = 0$$

$$\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x-y} = \lim_{(x,y)\to(0,0)} \frac{(x-y)(x+y)}{x-y} = \lim_{(x,y)\to(0,0)} (x+y) = 0+0 = 0$$

التمرين ١٢

أحسب :

$$J_1 = \int_0^2 |x - 1|\, dx \quad , \quad J_2 = \int_0^2 |x^2 - 3x + 2|\, dx$$

$$J_3 = \int_{\frac{1}{e}}^{e} \frac{|\ln x|}{x}\, dx \quad , \quad J_4 = \int_{-1}^{1} |e^x - 1|\, dx$$

الحل

$$J_1 = \int_0^2 |x - 1|\, dx$$

$$= \int_0^1 (1 - x)\, dx + \int_1^2 (x - 1)\, dx$$

$$= \left[x - \frac{x^2}{2}\right]_0^1 + \left[\frac{x^2}{2} - x\right]_1^2$$

$$= 1$$

إذن :

$$J_2 = \int_0^2 |x^2 - 3x + 2|\, dx$$

$$= \int_0^1 (x^2 - 3x + 2)\, dx + \int_1^2 -(x^2 - 3x + 2)\, dx$$

$$= \left[\frac{x^3}{3} - \frac{3}{2}x^2 + 2x\right]_0^1 - \left[\frac{x^3}{3} - \frac{3}{2}x^2 + 2x\right]_1^2$$

$$= 1$$

$$J_3 = \int_{\frac{1}{e}}^{e} \frac{|\ln x|}{x}\, dx$$

$$= \int_{\frac{1}{e}}^{1} \frac{-\ln x}{x}\, dx + \int_1^e \frac{\ln x}{x}\, dx$$

$$= \left[-\frac{1}{2}\ln^2 x\right]_{\frac{1}{e}}^{1} + \left[\frac{1}{2}\ln^2 x\right]_1^e$$

$$= 1$$

$$J_4 = \int_{-1}^{1} |e^x - 1|\, dx$$

$$= \int_{-1}^{0} (1 - e^x)\, dx + \int_0^1 (e^x - 1)\, dx$$

$$= \left[x - e^x\right]_{-1}^{0} + \left[e^x - x\right]_0^1$$

$$= e + e^{-1} - 2$$

Moutamadris.ma