

تقارین و حلولها

$$\frac{\pi}{5} = \frac{200}{12} = 40 \text{ gr}$$

$$\frac{\pi}{6} = \frac{180^\circ}{6} = 30^\circ$$

$$\frac{\tau}{6} = \frac{200}{6} = 33,33 \text{ gr}$$

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$\frac{\pi}{6}$	$\frac{\pi}{5}$	$\frac{\pi}{4}$	$\frac{\pi}{12}$	$\frac{\pi}{8}$	الرadian
30°	36°	45°	15°	22.5°	الدرجة
33,33gr	40°	50	16,66gr	25 gr	الgrad

تمرين 2:

- 1 - احسب بالرadian قياسات زوايا مثلث متساوي الأضلاع.

2 - احسب بالراديان قياسات زوايا مثلث قائم الزاوية ومتساوي الساقين.

الجواب :

- 1 - ليكن $\triangle ABC$ مثلثاً متساوياً الأضلاع. نعلم أن $\hat{A} = \hat{B} = \hat{C} = 60^\circ$. إذن: $\hat{A} = \hat{B} = \hat{C} = \frac{\pi}{3}$

2 - ليكن $\triangle EFG$ مثلثاً متساوياً الساقين وقائم الزاوية مثلثاً في E . نعلم أن $\hat{E} = 90^\circ$.

تمرين 1 :

اتم الجدول التالي :

$\frac{\pi}{6}$				$\frac{\pi}{8}$	الرadian
	36°		15°		الدرجة
		50			الغراد

الجواب :

$$\frac{\pi}{8} = \frac{180}{8} = 22^{\circ},5: \text{لدينا} *$$

$$\frac{\pi}{8} = \frac{200}{8} = 25 \text{ gr}$$

$$\frac{x}{\pi} = \frac{15}{180} : \text{لدينا} *$$

$$x = \frac{15\pi}{180} \quad \text{أي أن}$$

$$x = \frac{\pi}{12}$$

$$\frac{\pi}{12} = \frac{200}{12} \approx 16,66 \text{ gr} \text{ لدينا} *$$

$$50 \text{ gr} = \frac{200}{4} x = \frac{180^\circ}{4} = 45^\circ$$

$$50 \text{ gr} = \frac{\pi}{4}$$

$$\frac{x}{\pi} = \frac{36^\circ}{180}$$

$$\frac{x}{\pi} = \frac{1}{5}$$

$$x = -\frac{\pi}{5}$$

$$= 100\pi + \frac{4\pi}{5}$$

$$\frac{504\pi}{5} \equiv \frac{4\pi}{5} [2\pi] \quad \text{إذن :}$$

و منه الأفصول المحنبي الرئيسي لـ M هو $\frac{4\pi}{5}$

$$\text{لأن } \frac{4\pi}{5} \in]-\pi, \pi]$$

$$-\frac{277\pi}{4} = -\frac{280\pi + 3\pi}{4} \quad \text{ج - لدينا :}$$

$$= -70\pi + \frac{3\pi}{4}$$

$$-\frac{277\pi}{4} \equiv \frac{3\pi}{4} [2\pi] \quad \text{إذن [2\pi]}$$

إذن الأفصول المحنبي الرئيسي للنقطة M هو $\frac{3\pi}{4}$

$$\frac{3\pi}{4} \in]-\pi, \pi] \quad \text{لأن}$$

$$x = \frac{45\pi}{4} \quad \text{أ - لدينا :}$$

$$= \frac{48\pi - 3\pi}{4}$$

$$= 12\pi - \frac{3\pi}{4}$$

$$x = 12\pi + y \quad \text{إذن}$$

$$x \equiv y [2\pi] \quad \text{و منه}$$

إذن x و y أقصولان منحيان لنفس النقطة

ب - نفترض أن : $x \equiv y [2\pi]$ أي أنه يوجد k

من \mathbb{Z} حيث :

$$-\frac{123\pi}{5} = \frac{337\pi}{5} + 2k\pi \quad \text{أي أن}$$

$$-\frac{123\pi}{5} - \frac{337\pi}{5} = 2k\pi \quad \text{أي أن}$$

$$2k\pi = -\frac{500\pi}{5} \quad \text{و منه}$$

$$\hat{F} = \hat{G} = 45^\circ \quad \text{و}$$

$$\hat{E} = \frac{\pi}{2} \quad \text{إذن}$$

$$\hat{F} = \hat{G} = \frac{\pi}{4} \quad \text{و}$$

تمرين 3 :

1 - حدد الأفصول المحنبي الرئيسي للنقطة في الحالات التالية :

$$M\left(-\frac{99\pi}{7}\right) - \text{أ}$$

$$M\left(\frac{504\pi}{7}\right) - \text{ب}$$

$$M\left(-\frac{277\pi}{4}\right) - \text{ج}$$

2 - هل العددان x و y هما أقصولان منحيان لنفس النقطة على الدائرة المثلثية في الحالتين التاليتين :

$$x = -\frac{3\pi}{4} \quad \text{و} \quad x = \frac{45\pi}{4} - \text{أ}$$

$$x = \frac{337\pi}{5} \quad \text{و} \quad x = -\frac{123\pi}{5} - \text{ب}$$

الجواب :

$$-\frac{99\pi}{7} = -\frac{98\pi + \pi}{7} - 1 \quad \text{أ - لدينا}$$

$$= -14\pi - \frac{\pi}{7}$$

$$-\frac{99\pi}{7} \equiv -\frac{\pi}{7} [2\pi] \quad \text{إذن :}$$

$$-\frac{\pi}{7} \in]-\pi, \pi] \quad \text{و}$$

إذن الأفصول المحنبي الرئيسي لـ M هو $-\frac{\pi}{7}$

$$\frac{504\pi}{5} = \frac{500\pi + 4\pi}{5} \quad \text{ب - لدينا}$$

إذن $\frac{41\pi}{6} \equiv \frac{5\pi}{6} [2\pi]$
الأصول المثلثي الرئيسي لـ C هو $\frac{5\pi}{6}$

$$\begin{aligned} & \text{* نعتبر } D\left(\frac{25\pi}{4}\right) \\ & \frac{25\pi}{4} - \frac{24\pi + \pi}{4} \quad \text{لدينا} \\ & = 6\pi : \frac{\pi}{4} \end{aligned}$$

إذن $\frac{25\pi}{4} \equiv \frac{\pi}{4} [2\pi]$
الأصول المثلثي الرئيسي لـ D هو $\frac{\pi}{4}$

$$\begin{aligned} & \text{* نعتبر } B\left(-\frac{33\pi}{4}\right) \\ & \frac{33\pi}{4} - \frac{-32\pi - \pi}{4} \quad \text{لدينا} \\ & = -8\pi : \frac{\pi}{4} \end{aligned}$$

إذن $\frac{33\pi}{4} \equiv -\frac{\pi}{4} [2\pi]$
الأصول المثلثي الرئيسي لـ E هو $\frac{\pi}{4}$

$$\begin{aligned} & \text{نعتبر } F\left(\frac{169\pi}{3}\right) \\ & \frac{169\pi}{8} - \frac{168\pi + \pi}{8} \quad \text{لدينا} \end{aligned}$$

$$\begin{aligned} & = 21\pi : \frac{\pi}{8} \\ & = 21\pi + \pi + \frac{\pi}{8} \\ & \text{إذن } \frac{169\pi}{8} \equiv \pi + \frac{\pi}{8} [2\pi] \end{aligned}$$

$$\equiv -\pi + \frac{\pi}{8} [2\pi]$$

$$\equiv -\frac{7\pi}{8} [2\pi]$$

الأصول المثلثي الرئيسي لـ E هو $-\frac{7\pi}{8}$

$$2k\pi = -100\pi \quad \text{ومنه}$$

$$2k\pi = -100$$

$$2k = -100$$

$$k = -50 \quad \text{ومنه}$$

$$\text{وما أن } k \in \mathbb{Z} \text{ فإن } k \equiv y [2\pi] \text{ إذن } x \equiv y [2\pi]$$

أصولان منحيان لنفس النقطة.

تمرين 4:

مثل على دائرة مثلثية النقط ذات الأصول المثلثية التالية :

$$\frac{169\pi}{4}; -\frac{33\pi}{4}; \frac{25\pi}{4}; \frac{41\pi}{6}; \frac{8\pi}{3}; -\pi$$

الجواب:

* نعتبر $(-\pi) A'$ إذن الأصول المثلثي الرئيسي لـ A' هو π

$$\text{نعتبر } B\left(\frac{8\pi}{3}\right)$$

$$\frac{8\pi}{3} - \frac{6\pi + 2\pi}{3} = 2\pi + \frac{2\pi}{3} \quad \text{لدينا}$$

$$\frac{8\pi}{3} \equiv \frac{2\pi}{3} [2\pi] \quad \text{إذن}$$

الأصول المثلثي الرئيسي لـ B هو $\frac{2\pi}{3}$

$$\text{نعتبر } C\left(\frac{41\pi}{6}\right)$$

$$\frac{41\pi}{6} - \frac{36\pi + 5\pi}{6} \quad \text{لدينا}$$

$$= 6\pi : \frac{5\pi}{6}$$

$$(\overrightarrow{CB}, \overrightarrow{BD}) \equiv (-\overrightarrow{BC}, \overrightarrow{BD}) [2\pi] \quad \text{لدينا}$$

$$\equiv \pi + (\overrightarrow{BC}, \overrightarrow{BD}) [2\pi]$$

$$\equiv \pi + \frac{\pi}{3} [2\pi]$$

$$\equiv \frac{4\pi}{3} [2\pi]$$

$$\equiv \frac{6\pi - 2\pi}{3} [2\pi]$$

$$\equiv -\frac{2\pi}{3} [2\pi]$$

إذن القياس الرئيسي للزاوية $(\overrightarrow{CB}, \overrightarrow{BD})$

$$-\frac{2\pi}{3} \quad \text{هو :}$$

لدينا :

$$(\overrightarrow{BA}, \overrightarrow{BD}) \equiv (\overrightarrow{BA}, \overrightarrow{BC}) + (\overrightarrow{BC}, \overrightarrow{BD}) [2\pi]$$

$$\equiv \frac{\pi}{4} + \frac{\pi}{3} [2\pi]$$

$$\equiv \frac{7\pi}{12} [2\pi]$$

إذن القياس الرئيسي للزاوية $(\overrightarrow{BA}, \overrightarrow{BD})$

$$\frac{7\pi}{12} \quad \text{هو :}$$

تمرين 6:

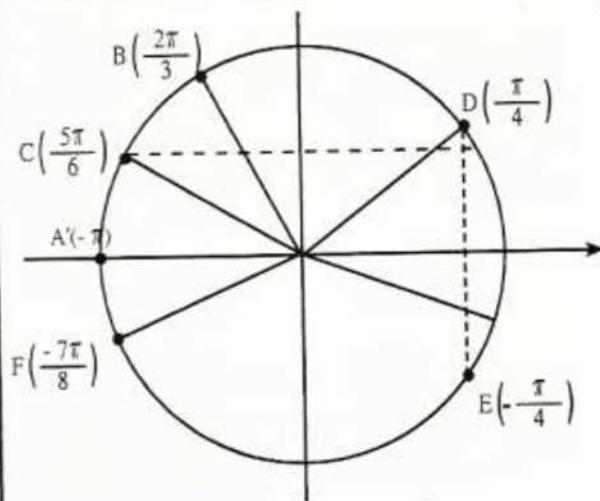
ABC مثلث بين أن :

$$(\overrightarrow{AB}, \overrightarrow{AC}) + (\overrightarrow{CA}, \overrightarrow{CB}) + (\overrightarrow{BC}, \overrightarrow{BA}) \equiv \pi [2\pi]$$

الجواب :

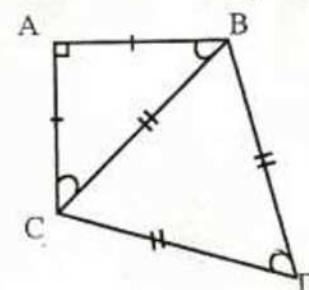
$$(\overrightarrow{BC}, \overrightarrow{BA}) + (\overrightarrow{CA}, \overrightarrow{CB}) + (\overrightarrow{AB}, \overrightarrow{AC}) \quad \text{لدينا *}$$

$$\equiv (\overrightarrow{AB}, \overrightarrow{AC}) + (-\overrightarrow{AC}, -\overrightarrow{BC}) + (\overrightarrow{BC}, \overrightarrow{BA}) [2\pi]$$



تمرين 5:

نعتبر الشكل التالي :



اعط القياس الرئيسي لكل من الزوايا التالية :

$$, (\overrightarrow{AB}, \overrightarrow{AC}), (\overrightarrow{DC}, \overrightarrow{DB}), (\overrightarrow{BA}, \overrightarrow{BC})$$

$$(\overrightarrow{BA}, \overrightarrow{BD}), (\overrightarrow{CB}, \overrightarrow{BD})$$

الجواب :

$$(\overrightarrow{BA}, \overrightarrow{BC}) \equiv -\frac{\pi}{4} [2\pi] \quad \text{لدينا *}$$

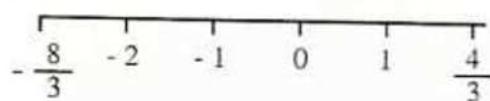
القياس الرئيسي للزاوية $(\overrightarrow{BA}, \overrightarrow{BC})$ هو $-\frac{\pi}{4}$

$$(\overrightarrow{DC}, \overrightarrow{DB}) \equiv -\frac{\pi}{3} [2\pi] \quad \text{لدينا :}$$

إذن القياس الرئيسي للزاوية $(\overrightarrow{DC}, \overrightarrow{DB})$ هو $-\frac{\pi}{3}$

$$(\overrightarrow{AB}, \overrightarrow{AC}) \equiv -\frac{\pi}{2} [2\pi] \quad \text{لدينا *}$$

إذن القياس الرئيسي $(\overrightarrow{AB}, \overrightarrow{AC})$ هو $-\frac{\pi}{2}$



ـ 1, 0, -1, -2 فـإن $k \in \mathbb{Z}$ تأخذ القيم

$$M_0\left(\frac{\pi}{3}\right) \quad \text{إذن } k=0$$

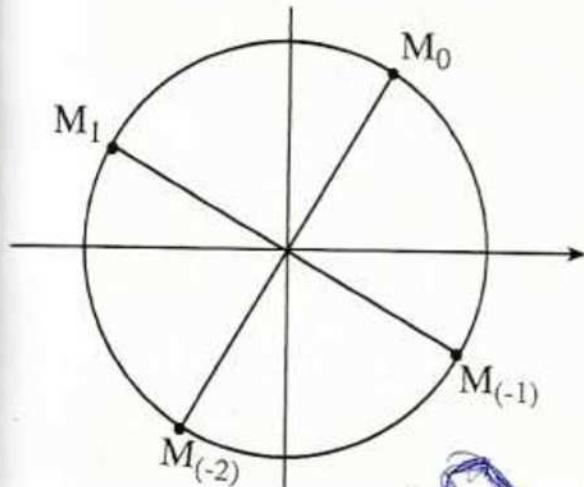
$$M_1\left(\frac{5\pi}{6}\right) \quad \text{إذن } k=1$$

$$M_{-1}\left(-\frac{\pi}{6}\right) \quad \text{إذن } k=-1$$

$$M_{-2}\left(-\frac{2\pi}{3}\right) \quad \text{إذن } k=-2$$

لتمثيل النقط M_k يكفي أن تمثيل النقط .

M_{-2}, M_{-1}, M_1, M_0



تمرين 8 :

ليكن x من المجموعة \mathbb{R} بسط مailyi :

$$A(x) = 2\sin^2(x) + 3\cos^2(x) - 1$$

$$B(x) = (\cos x + \sin x)^2 - 1$$

$$C(x) = \cos^2 x - \cos^2 x \cdot \sin^2 x$$

$$D(x) = (2\cos x + \sin x)^2 + (\cos x - 2\sin x)^2$$

$$E(x) = \cos^5 x + \cos^3 x \cdot \sin^2 x$$

$$F(x) = \cos^4 x - \cos^2 x + \sin^2 x - \sin^4 x$$

$$G(x) = \cos^6 x + \sin^6 x + 3\sin^2 x \cdot \cos^2 x$$

$$\equiv (\overrightarrow{AB}, \overrightarrow{AC}) + (\overrightarrow{AC}, \overrightarrow{BC}) + (\overrightarrow{BC}, \overrightarrow{BA}) [2\pi]$$

$$\equiv (\overrightarrow{AB}, \overrightarrow{BC}) + (\overrightarrow{BC}, \overrightarrow{BA}) [2\pi]$$

$$\equiv (\overrightarrow{AB}, \overrightarrow{BA}) [2\pi]$$

$$\equiv (\overrightarrow{AB}, \overrightarrow{AB}) [2\pi]$$

$$\equiv \pi + (\overrightarrow{AB}, \overrightarrow{AB}) [2\pi]$$

$$\equiv \pi + 0 [2\pi]$$

إذن :

$$(\overrightarrow{AB}, \overrightarrow{AC}) + (\overrightarrow{CA}, \overrightarrow{CB}) + (\overrightarrow{BC}, \overrightarrow{BA}) \equiv \pi [2\pi]$$

تمرين 7 :

مثل على دائرة مثلثية النقط M_k التي أفادتها
المنحنية هي الأعداد :

$$k \in \mathbb{Z}, \frac{\pi}{3} + \frac{k\pi}{2}$$

الجواب :

$$k \in \mathbb{Z}, B \left(\frac{\pi}{3} + \frac{k\pi}{2} \right) * \quad \text{لدينا}$$

لنحدد قيم k التي من أجلها يكون $\frac{\pi}{3} + \frac{k\pi}{2}$
قياساً رئيسياً لـ M_k .

$$-\pi < \frac{k\pi}{2} + \frac{\pi}{3} < \pi \quad \text{إذن}$$

$$-1 < \frac{1}{3} + \frac{k}{2} < 1 \quad \text{أي أن}$$

$$-\frac{4}{3} < \frac{k}{2} < \frac{2}{3} \quad \text{إذن}$$

$$-\frac{8}{3} < k < \frac{4}{3} \quad \text{أي أن}$$

$$C(x) = \cos^3 x \quad : \quad \text{إذن} * \quad \text{لدينا}$$

$$\begin{aligned} F(x) &= \cos^4 x - \cos^2 x + \sin^2 x - \sin^4 x \\ &= \cos^2 x (\cos^2 x - 1) + \sin^2 x (1 - \sin^2 x) \\ &= \cos^2 x \cdot (-\sin^2 x) + \sin^2 x \cdot \cos^2 x \\ &= -\cos^2 x \cdot \sin^2 x + \sin^2 x \cdot \cos^2 x \\ &= 0 \end{aligned}$$

$$F(x) = 0 \quad : \quad \text{إذن} * \quad \text{لدينا}$$

$$\begin{aligned} G(x) &= \cos^6 x + \sin^6 x + 3\sin^2 x \cdot \cos^2 x \\ &= (\cos^2 x)^3 + (\sin^2 x)^3 + 3\sin^2 x \cdot \cos^2 x \\ &= (\cos^2 x + \sin^2 x)(\cos^4 x - \cos^2 x \cdot \sin^2 x + \sin^4 x) + 3\sin^2 x \cdot \cos^2 x \\ &= \cos^4 x - \cos^2 x \cdot \sin^2 x + \sin^4 x + 3\sin^2 x \cdot \cos^2 x \\ &= \cos^4 x + \sin^4 x + 2\sin^2 x \cdot \cos^2 x \\ &= (\cos^2 x)^2 + (\sin^2 x)^2 + 2\sin^2 x \cdot \cos^2 x \\ &= (\cos^2 x + \sin^2 x)^2 \\ &= 1^2 \\ &= 1 \end{aligned}$$

$$F(x) = 1 \quad : \quad \text{إذن}$$

تمرين 9

- ليكن x من المجال $\left[0, \frac{\pi}{2}\right]$
 . $\tan x$ و $\cos x$ احسب $\sin x = \frac{\sqrt{5}}{4}$
 $x \in \left[\frac{\pi}{2}, \pi\right]$ - إذا علمت أن: 2

الجواب:

$$\begin{aligned} A(x) &= 2\sin^2(x) + 3\cos^2(x) - 1 \quad * \quad \text{لدينا} \\ &= 2\sin^2 x + 3(1 - \sin^2 x) - 1 \\ &= 2\sin^2 x + 3 - 3\sin^2 x - 1 \\ &= 2 - \sin^2 x \end{aligned}$$

$$\begin{aligned} A(x) &= 2 - \sin^2 x \quad : \quad \text{إذن} * \quad \text{لدينا} \\ B(x) &= (\cos x + \sin x)^2 - 1 \quad * \quad \text{لدينا} \\ &= \cos^2 x + \sin^2 x + 2\sin x \cos x - 1 \\ &= 1 + 2\sin x \cos x - 1 \\ &= 2\sin x \cos x \end{aligned}$$

$$\begin{aligned} B(x) &= 2\sin x \cos x \quad : \quad \text{إذن} * \quad \text{لدينا} \\ C(x) &= \cos^2 x - \cos^2 x \cdot \sin^2 x \quad * \quad \text{لدينا} \\ &= \cos^2 x (1 - \sin^2 x) \\ &= \cos^2 x \cdot \cos^2 x \end{aligned}$$

$$\begin{aligned} C(x) &= \cos^4 x \quad : \quad \text{إذن} * \quad \text{لدينا} \\ D(x) &= (2\cos x + \sin x)^2 + (\cos x - 2\sin x)^2 \\ &= 4\cos^2 x + \sin^2 x + 4\cos x \sin x + \cos^2 x \\ &\quad + 4\sin^2 x - 4\sin x \cos x \\ &= 5\cos^2 x + 5\sin^2 x \\ &= 5(\cos^2 x + \sin^2 x) \\ &= 5 \times 1 \end{aligned}$$

$$\begin{aligned} D(x) &= 5 \quad : \quad \text{إذن} * \quad \text{لدينا} \\ E(x) &= \cos^5 x + \cos^3 x \cdot \sin^2 x \quad * \quad \text{لدينا} \\ &= \cos^3 x (\cos^2 x + \sin^2 x) \\ &= \cos^3 x \cdot 1 \end{aligned}$$

$$\sin^2 x + \left(-\frac{2}{3}\right)^2 = 1 \quad \text{أي أن}$$

$$\sin^2 x = 1 - \frac{4}{9}$$

$$\sin^2 x = \frac{5}{9}$$

$\sin x = -\frac{\sqrt{5}}{3}$ أو $\sin x = \frac{\sqrt{5}}{3}$ ومنه

و بما أن $\sin x \geq 0$ فإن

نعلم أن لكل x من $\left[\frac{\pi}{2}, \pi\right]$

$$\tan x = \frac{\sin x}{\cos x} = \frac{\frac{\sqrt{5}}{3}}{-\frac{2}{3}} = -\frac{\sqrt{5}}{2}$$

$\tan x = -\frac{\sqrt{5}}{2}$ إذن :

$\tan \alpha = \sqrt{7}$: لدينا - 3

$$\cos^2 \alpha = \frac{1}{1 + \tan^2 \alpha} \quad \text{نعلم أن}$$

$$= \frac{1}{1 + (\sqrt{7})^2}$$

$$= \frac{1}{8}$$

$\cos \alpha = \sqrt{\frac{1}{8}}$ أو $\cos \alpha = -\sqrt{\frac{1}{8}}$ إذن

$\cos \alpha = \frac{1}{2\sqrt{2}}$ أو $\cos \alpha = -\frac{1}{2\sqrt{2}}$

$\cos \alpha = \frac{\sqrt{2}}{4}$ أو $\cos \alpha = -\frac{\sqrt{2}}{4}$

بما أن $\cos \alpha < 0$ فإن

. $\tan x$ و $\sin x$: فاحسب $\cos x = -\frac{2}{3}$ و

- إذا علمت أن $\alpha \in \left[-\pi, -\frac{\pi}{2}\right]$

$\tan \alpha = \sqrt{7}$ و

. فاحسب $\sin \alpha$ و $\cos \alpha$:

$$\cos \frac{5\pi}{12} = \frac{\sqrt{6} - \sqrt{2}}{4} \quad - \text{إذا علمت أن :}$$

. $\tan \frac{5\pi}{12}$ ثم $\cos \frac{5\pi}{12}$ فاحسب

الجواب :

$$\sin x = \frac{\sqrt{5}}{4} \quad 1 - \text{لدينا :}$$

نعلم أن 1

$$\cos^2 x + \sin^2 x = 1 \quad \text{أي أن}$$

$$\cos^2 x = 1 - \frac{5}{16} \quad \text{إذن}$$

$$\cos^2 x = \frac{11}{16} \quad \text{أي أن}$$

$$\cos x = \frac{\sqrt{11}}{4} \quad \text{أو } \cos x = -\frac{\sqrt{11}}{4} \quad \text{و منه}$$

. $\cos x \geq 0$ فإن $x \in \left[0, \frac{\pi}{2}\right]$

$$\cos x = \frac{\sqrt{11}}{4} \quad \text{و منه}$$

. $\left[0, \frac{\pi}{2}\right] \quad \text{نعلم أن لكل } x \text{ من}$

$$\tan x = \frac{\sin x}{\cos x}$$

$$= \frac{\frac{\sqrt{5}}{4}}{\frac{\sqrt{11}}{4}} = \frac{\sqrt{5}}{\sqrt{11}} = \frac{\sqrt{55}}{11}$$

$$\cdot \cos x = -\frac{2}{3} \quad \text{لدينا - 2}$$

نعلم أن 1

$$\sin \frac{5\pi}{12} > 0 \text{ لأن } 0 < \frac{5\pi}{12} < \frac{\pi}{2}$$

$$\sin \frac{5\pi}{12} = \frac{\sqrt{6} + \sqrt{2}}{4} \quad \text{ومنه}$$

$$\tan \frac{5\pi}{12} = \frac{\sin \frac{5\pi}{12}}{\cos \frac{5\pi}{12}} \quad \text{لدينا}$$

$$\begin{aligned} &= \frac{\frac{\sqrt{6} + \sqrt{2}}{4}}{\frac{\sqrt{6} - \sqrt{2}}{4}} \\ &= \frac{\sqrt{6} + \sqrt{2}}{\sqrt{6} - \sqrt{2}} \\ &= \frac{(\sqrt{6} + \sqrt{2})^2}{6 - 2} \\ &= \frac{8 + 2\sqrt{12}}{4} \\ &= \frac{8 + 4\sqrt{3}}{4} \\ &= \frac{4(2 + \sqrt{3})}{4} \end{aligned}$$

$$\tan \frac{5\pi}{12} = 2 + \sqrt{3} \quad \text{إذن :}$$

$$\tan \alpha = \frac{\sin x}{\cos x} \quad \text{نعلم أن}$$

$$\sin \alpha = (\cos \alpha) \cdot (\tan \alpha) \quad \text{إذن}$$

$$\sin \alpha = \left(\frac{-\sqrt{2}}{4} \right) \times \sqrt{7} = -\frac{\sqrt{14}}{4}$$

$$\cos \frac{5\pi}{12} = \frac{\sqrt{6} - \sqrt{2}}{2} : \text{لدينا} - 4$$

$$\cos^2 \frac{5\pi}{12} + \sin^2 \frac{5\pi}{12} = 1 \quad \text{نعلم أن}$$

$$\sin^2 \frac{5\pi}{12} + \left(\frac{\sqrt{6} - \sqrt{2}}{4} \right)^2 = 1 \quad \text{إذن}$$

$$\sin^2 \frac{5\pi}{12} = 1 - \left(\frac{\sqrt{6} - \sqrt{2}}{4} \right)^2$$

$$= 1 - \frac{8 - 2\sqrt{12}}{16}$$

$$= \frac{16 - 8 + 2\sqrt{12}}{16}$$

$$= \frac{8 + 2\sqrt{12}}{16}$$

$$= \left(\frac{\sqrt{6} + \sqrt{2}}{4} \right)^2$$

$$\sin \frac{5\pi}{12} = -\frac{\sqrt{6} + \sqrt{2}}{4} \quad \text{إذن}$$

$$\sin \frac{5\pi}{12} = \frac{\sqrt{6} + \sqrt{2}}{4} \quad \text{أو}$$

تمرين 10

1 - احسب التعبيرات التالية :

$$A = \sin^6 x + \cos^6 x - 2\sin^4 x - \cos^4 x + \sin^2 x$$

$$B = 2(\cos^6 x + \sin^6 x) - 3(\cos^4 x + \sin^4 x)$$

$$C = \sin^8 x + \cos^8 x + 6\sin^4 x \cdot \cos^4 x + 4\sin^2 x \cos^2 x (\sin^4 x + \cos^4 x)$$

$$D = \sqrt{\sin^4 x + 4\cos^2 x} + \sqrt{\cos^4 x + 4\sin^2 x}$$

$$E = \frac{\cos x}{1 + \sin x} + \frac{\sin x}{1 + \cos x} + \frac{(1 - \sin x)(1 - \cos x)}{\sin x \cdot \cos x}$$

الجواب :

$$A = \sin^6 x + \cos^6 x - 2\sin^4 x - \cos^4 x + \sin^2 x$$

1 - لدينا :

$$= (\sin^2 x)^3 + (\cos^2 x)^3 - 2\sin^4 x - \cos^4 x + \sin^2 x$$

$$= (\sin^2 x + \cos^2 x)(\sin^4 x + \cos^4 x - \sin^2 x \cos^2 x) - 2\sin^4 x - \cos^4 x + \sin^2 x$$

$$= \sin^4 x + \cos^4 x - \sin^2 x \cos^2 x - 2\sin^4 x - \cos^4 x + \sin^2 x$$

$$= -\sin^4 x + \sin^2 x - \sin^2 x \cdot \cos^2 x$$

$$= \sin^2 x (1 - \sin^2 x) - \sin^2 x \cdot \cos^2 x$$

$$= \sin^2 x \cdot \cos^2 x - \sin^2 x \cdot \cos^2 x$$

$$= 0$$

$A = 0$

$$B = 2(\cos^6 x + \sin^6 x) - 3(\cos^4 x + \sin^4 x)$$

$$= 2(\cos^2 x + \sin^2 x)(\cos^4 x + \sin^4 x - \sin x \cos x) - 3(\cos^4 x + \sin^4 x)$$

$$= 2\cos^4 x + 2\sin^4 x - 2\sin^2 x \cdot \cos^2 x - 3\cos^4 x - 3\sin^4 x$$

$$= -\cos^4 x - \sin^4 x - 2\sin^2 x \cdot \cos^2 x$$



$$= -(\cos^4 x + \sin^4 x + 2\sin^2 x \cdot \cos^2 x)$$

$$= -(\cos^2 x + \sin^2 x)^2 = -1$$

$$\boxed{B = -1}$$

$$C = \sin^8 x + \cos^8 x + 6\sin^4 x \cdot \cos^4 x + 4\sin^2 x \cos^2 x (\sin^4 x + \cos^4 x)$$

$$= (\sin^4 x + \cos^4 x)^2 - 2\sin^4 x \cos^4 x + 6\sin^4 x \cos^4 x + 4\sin^2 x \cos^2 x (\sin^4 x + \cos^4 x)$$

$$= (\sin^4 x + \cos^4 x)^2 + 4\sin^2 x \cos^2 x (\sin^4 x + \cos^4 x) + 4\sin^4 x \cdot \cos^4 x$$

$$= (\sin^4 x + \cos^4 x)(\sin^4 x + \cos^4 x + 2\sin^2 x \cdot \cos^2 x) + 2\sin^2 x \cos^2 x (\sin^4 x + \cos^4 x) + 4\sin^4 x \cdot \cos^4 x$$

$$= (\sin^4 x + \cos^4 x)(\sin^2 x + \cos^2 x)^2 + 2\sin^6 x \cos^2 x + 2\sin^2 x \cdot \cos^6 x + 4\sin^4 x \cdot \cos^4 x$$

$$= \sin^4 x + \cos^4 x + (2\sin^6 x \cdot \cos^2 x + 2\sin^4 x \cos^4 x) + (2\sin^2 x \cos^6 x + 2\sin^4 \cos^4 x)$$

$$= \sin^4 x + \cos^4 x + 2\sin^4 x \cos^2 x + 2\cos^4 x \cdot \sin^2 x (\cos^2 x + \sin^2 x)$$

$$= \sin^4 x + \cos^4 x + 2\cos^2 x \cdot \sin^2 x (\sin^2 x + \cos^2 x)$$

$$= (\sin^2 x)^2 + (\cos^2 x)^2 + 2\sin^2 x \cdot \cos^2 x$$

$$= (\sin^2 x + \cos^2 x)^2$$

$$= 1^2$$

$$= 1$$

$$\boxed{C = 1}$$

$$D = \sqrt{\sin^4 x + 4\cos^2 x} + \sqrt{\cos^4 x + 4\sin^2 x}$$

$$= \sqrt{\sin^4 x + 4(1 - \sin^2 x)} + \sqrt{\cos^4 x + 4(1 - \cos^2 x)}$$

$$= \sqrt{\sin^4 x - 4\sin^2 x + 4} + \sqrt{\cos^4 x - 4\cos^2 x + 4}$$

$$= \sqrt{(\sin^2 x - 2)^2} + \sqrt{(\cos^2 x - 2)^2}$$

$$= |\sin^2 x - 2| + |\cos^2 x - 2|$$

$$= 2 - \sin^2 x + 2 - \cos^2 x$$

$$-1 \leq \sin x \leq 1 , \quad -1 \leq \cos x \leq 1 \quad \text{لأن}$$

$$0 \leq \sin^2 x \leq 1 , \quad 0 \leq \cos^2 x \leq 1 \quad \text{ومنه}$$

$$\sin^2 x - 2 \leq 0 , \quad \cos^2 x - 2 \leq 0 \quad \text{إذن}$$



$$D = 4 - (\sin^2 x + \cos^2 x) \quad \text{إذن}$$

$$= 4 - 1$$

$$= 3$$

$D = 3$

$$\begin{aligned}
 E &= \frac{\cos x}{1 + \sin x} + \frac{\sin x}{1 + \cos x} + \frac{(1 - \sin x)(1 - \cos x)}{\sin x \cdot \cos x} \\
 &= \frac{\cos x (1 - \sin x)}{(1 + \sin x)(1 - \sin x)} + \frac{\sin x (1 - \sin x)}{(1 + \cos x)(1 - \cos x)} + \frac{1 - \cos x - \sin x + \cos x \sin x}{\sin x \cdot \cos x} \\
 &= \frac{\cos x (1 - \sin x)}{1 - \sin^2 x} + \frac{\sin x (1 - \cos x)}{1 - \cos^2 x} + \frac{1 - \cos x - \sin x + \sin x \cos x}{\sin x \cdot \cos x} \\
 &= \frac{\cos x (1 - \sin x)}{\cos^2 x} + \frac{\sin x (1 - \cos x)}{\sin^2 x} + \frac{1 - \cos x - \sin x + \sin x \cos x}{\sin x \cdot \cos x} \\
 &= \frac{1 - \sin x}{\cos x} + \frac{1 - \sin x}{\sin x} + \frac{1 - \cos x - \sin x + \sin x \cos x}{\sin x \cdot \cos x} \\
 &= \frac{\sin x (1 - \sin x) + \cos x (1 - \cos x) + 1 - \cos x - \sin x + \sin x \cos x}{\sin x \cdot \cos x} \\
 &= \frac{\sin x - \sin^2 x + \cos x - \cos^2 x + 1 - \cos x - \sin x + \sin x \cos x}{\sin x \cdot \cos x} \\
 &= \frac{-(\sin^2 x + \cos^2 x) + 1 + \sin x \cdot \cos x}{\sin x \cdot \cos x} \\
 &= \frac{\sin x \cos x}{\sin x \cos x} = 1
 \end{aligned}$$

تمرين 11:

1 - ليكن x من $\left[/ \{0\} \right] - \pi, \pi$ ، $x \neq -\frac{\pi}{2}$ و $x \neq \frac{\pi}{2}$

$$\frac{1}{\tan^2(x)} - \cos^2(x) = \cos^2(x) \times \frac{1}{\tan^2(x)} : \text{بين أن}$$

2 - ليكن x و y من $[-\pi, \pi]$ و يخالفان $\frac{\pi}{2}$ و $-\frac{\pi}{2}$

$$\sin^2 x - \sin^2 y = \frac{1}{1 + \tan^2 y} - \frac{1}{1 + \tan^2 x} : \text{بين أن}$$

الجواب :

$$\begin{aligned}
 A &= 2\sin x \cos x (1 - 2\sin^2 x) \quad \text{لدينا} : \\
 &= 2(\tan x \cos x) \cdot \cos x (1 - 2(1 - \cos^2 x)) \\
 &= 2\tan x \cdot \cos^2 x (2\cos^2 x - 1) \\
 &= 2\tan x \left(\frac{1}{1 + \tan^2 x} \right) \left(\frac{2}{1 + \tan^2 x} - 1 \right) \\
 &= 2\tan x \left(\frac{1}{1 + \tan^2 x} \right) \left(\frac{1 - \tan^2 x}{1 + \tan^2 x} \right) \\
 &= \frac{2(\tan x)(1 - \tan^2 x)}{(1 + \tan^2 x)^2} \\
 A &= \boxed{\frac{2(\tan x)(1 - \tan^2 x)}{(1 + \tan^2 x)^2}} \quad \text{إذن}
 \end{aligned}$$

$$\begin{aligned}
 B &= \frac{\sin^3 x - \cos^3 x}{\sin x - \cos x} \quad \text{لدينا *} \\
 &= \frac{(\sin x - \cos x)(\sin^2 x + \sin x \cos x + \cos^2 x)}{\sin x - \cos x} \\
 &= \sin^2 x + \sin x \cdot \cos x + \cos^2 x \\
 &= 1 - \cos^2 x + \sin x \cdot \cos x + \cos^2 x \\
 &= 1 - \sin x \cdot \cos x \\
 &= 1 - (\tan x)(\cos x) \cdot \cos x \\
 &= 1 - \tan x \times \cos^2 x \\
 &= 1 - \tan x \times \frac{1}{1 + \tan^2 x} \\
 &= 1 - \frac{\tan x}{1 + \tan^2 x} \\
 &= \frac{1 + \tan^2 x - \tan x}{1 + \tan^2 x} \\
 B &= \boxed{\frac{\tan^2 x - \tan x + 1}{\tan^2 x + 1}} \quad \text{إذن}
 \end{aligned}$$

الجواب : - 1

$$\begin{aligned}
 \frac{1}{\tan^2(x)} - \cos^2 x &= \frac{1}{\frac{\sin^2(x)}{\cos^2(x)}} - \cos^2 x \\
 &= \frac{\cos^2 x}{\sin^2 x} - \cos^2 x \\
 &= \cos^2 x \left(\frac{1}{\sin^2 x} - 1 \right) \\
 &= \cos^2 x \left(\frac{1 - \sin^2 x}{\sin^2 x} \right) \\
 &= \cos^2 x \left(\frac{\cos^2 x}{\sin^2 x} \right) \quad \text{إذن} \\
 (\cos^2 x) \times \frac{1}{\tan^2 x} &= \frac{1}{\tan^2(x)} - \cos^2(x) \\
 &\quad \text{لدينا 2}
 \end{aligned}$$

$$\begin{aligned}
 \sin^2 x - \sin^2 y &= 1 - \cos^2 x - (1 - \cos^2 y) \\
 &= -\cos^2 x + \cos^2 y \\
 &= \cos^2 y - \cos^2 x
 \end{aligned}$$

$$= \frac{1}{1 + \tan^2 y} - \frac{1}{1 + \tan^2 x}$$

تمرين 12:

$$1 - \text{لتكن } x \text{ من المجال } [0, \pi] / \left\{ \frac{\pi}{2} \right\}$$

حدد بدلالة $\tan(x)$ مايلي :

$$\begin{aligned}
 A &= 2\sin x \cos x (1 - 2\sin^2 x) \\
 B &= \frac{\sin^3 x - \cos^3 x}{\sin x - \cos x}; \quad x \neq \frac{\pi}{4} \\
 C &= \cos^4 x - \sin^4 x + \cos^2 x - \sin^2 x
 \end{aligned}$$

$$\begin{aligned}
 &= \cos\left(4\pi + \frac{2\pi}{3}\right) + \sin\left(4\pi - \frac{\pi}{6}\right) \\
 &- 2\sin\left(4\pi + \frac{\pi}{2}\right) \\
 &= \cos\left(\frac{2\pi}{3}\right) + \sin\left(-\frac{\pi}{6}\right) - 2\sin\left(\frac{\pi}{2}\right) \\
 &= \cos\left(\pi - \frac{\pi}{3}\right) - \sin\left(\frac{\pi}{6}\right) - 2\sin\left(\frac{\pi}{2}\right) \\
 &= -\cos\frac{\pi}{3} - \sin\frac{\pi}{6} - 2\sin\frac{\pi}{2} \\
 &= -\frac{1}{2} - \frac{1}{2} - 2 \times 1 \\
 &= -1 - 2
 \end{aligned}$$

A = -3

إذن

$$\begin{aligned}
 B &= \cos\left(\frac{3\pi}{4}\right) \times \sin\left(\frac{4\pi}{3}\right) \cos\left(\frac{7\pi}{6}\right) \times \\
 &\sin\left(\frac{5\pi}{4}\right) \\
 &= \cos\left(\pi - \frac{\pi}{4}\right) \times \sin\left(\pi + \frac{\pi}{3}\right) \times \\
 &\cos\left(\pi - \frac{\pi}{6}\right) \times \sin\left(\pi + \frac{\pi}{4}\right) \\
 &= \left(-\cos\frac{\pi}{3}\right) \times \left(-\sin\frac{\pi}{3}\right) \times \left(-\cos\frac{\pi}{6}\right) \times \left(-\sin\frac{\pi}{4}\right) \\
 &= \left(\cos\frac{\pi}{4}\right) \times \left(\sin\frac{\pi}{3}\right) \times \left(\cos\frac{\pi}{6}\right) \times \left(\sin\frac{\pi}{4}\right) \\
 &= \frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{2} \times \frac{1}{2} \times \frac{\sqrt{2}}{2} \\
 &= \frac{2 \times \sqrt{2}}{2 \times 8} = \frac{\sqrt{3}}{8}
 \end{aligned}$$

B = $\frac{\sqrt{3}}{8}$

$$\begin{aligned}
 C &= \tan\left(\frac{2\pi}{3}\right) \times \tan\left(\frac{5\pi}{4}\right) \times \tan\left(\frac{5\pi}{6}\right) \\
 &= \tan\left(\pi - \frac{\pi}{3}\right) \times \tan\left(\pi + \frac{\pi}{4}\right) \times \tan\left(\pi + \frac{\pi}{6}\right) \\
 &= -\tan\left(\frac{\pi}{3}\right) \times \tan\left(\frac{\pi}{4}\right) \times \tan\left(\frac{\pi}{6}\right) \\
 &= \sqrt{3} \times 1 \times \frac{\sqrt{3}}{3} \\
 &= \frac{-3}{3} \\
 &= -1
 \end{aligned}$$

C = -1

إذن

: لدينا *

$$\begin{aligned}
 C &= \cos^4 x - \sin^4 x + \cos^2 x - \sin^2 x \\
 &= (\cos^2 x)^2 - (\sin^2 x)^2 + \cos^2 x - \sin^2 x \\
 &= (\cos^2 x + \sin^2 x)(\cos^2 x - \sin^2 x) + \cos^2 x - \sin^2 x \\
 &= \cos^2 x - \sin^2 x + \cos^2 x - \sin^2 x \\
 &= 2(\cos^2 x - \sin^2 x) \\
 &= 2(\cos^2 x - 1 + \cos^2 x) \\
 &= 2(2\cos^2 x - 1) \\
 &= 2\left(\frac{2}{1 + \tan^2 x} - 1\right) \\
 &= 2\left(\frac{2 - 1 - \tan^2 x}{1 + \tan^2 x}\right)
 \end{aligned}$$

C = $\frac{2(1 - \tan^2 x)}{1 + \tan^2 x}$

إذن

تمرين 13:

: احسب ما يلي

$$\begin{aligned}
 A &= \cos\left(\frac{14\pi}{3}\right) + \sin\left(\frac{23\pi}{6}\right) - 2\sin\left(\frac{9\pi}{2}\right) \\
 B &= \cos\left(\frac{3\pi}{4}\right) \times \sin\left(\frac{4\pi}{3}\right) \cos\left(\frac{7\pi}{6}\right) \times \sin\left(\frac{5\pi}{4}\right) \\
 C &= \tan\left(\frac{2\pi}{3}\right) \times \tan\left(\frac{5\pi}{4}\right) \times \tan\left(\frac{5\pi}{6}\right)
 \end{aligned}$$

الجواب :

$$\begin{aligned}
 A &= \cos\left(\frac{12\pi + 2\pi}{3}\right) + \sin\left(\frac{24\pi - \pi}{6}\right) \\
 &- 2\sin\left(\frac{8\pi + \pi}{2}\right)
 \end{aligned}$$

تمرين 14:

احسب مايلي :

$$A = \cos\left(\frac{\pi}{7}\right) + \cos\left(\frac{2\pi}{7}\right) + \cos\left(\frac{5\pi}{7}\right) + \cos\left(\frac{6\pi}{7}\right)$$

$$B = \tan\left(\frac{\pi}{7}\right) + \tan\left(\frac{2\pi}{7}\right) + \tan\left(\frac{5\pi}{7}\right) + \tan\left(\frac{6\pi}{7}\right)$$

$$C = \cos^2\left(\frac{\pi}{12}\right) + \cos^2\left(\frac{5\pi}{12}\right) + \cos^2\left(\frac{7\pi}{12}\right) + \cos^2\left(\frac{11\pi}{12}\right)$$

الجواب :

لدينا :

$$A = \cos\left(\frac{\pi}{7}\right) + \cos\left(\frac{2\pi}{7}\right) + \cos\left(\frac{5\pi}{7}\right) + \cos\left(\frac{6\pi}{7}\right)$$

$$= \cos\left(\frac{\pi}{7}\right) + \cos\left(\frac{2\pi}{7}\right) + \cos\left(\frac{7\pi - 2\pi}{7}\right) + \cos\left(\frac{7\pi - 2\pi}{7}\right)$$

$$= \cos\left(\frac{\pi}{7}\right) + \cos\left(\frac{2\pi}{7}\right) + \cos\left(\pi - \frac{2\pi}{7}\right) + \cos\left(\pi - \frac{\pi}{7}\right)$$

$$= \cos\left(\frac{\pi}{7}\right) + \cos\left(\frac{2\pi}{7}\right) - \cos\left(\frac{2\pi}{7}\right) - \cos\left(\frac{\pi}{7}\right)$$

$$= 0$$

$$A = 0$$

إذن

$$B = \tan\left(\frac{\pi}{7}\right) + \tan\left(\frac{2\pi}{7}\right) + \tan\left(\frac{5\pi}{7}\right) + \tan\left(\frac{6\pi}{7}\right)$$

$$- \tan\left(\frac{\pi}{7}\right) + \tan\left(\frac{2\pi}{7}\right) + \tan\left(\pi - \frac{2\pi}{7}\right) + \tan\left(\pi - \frac{\pi}{7}\right)$$

$$- \tan\left(\frac{\pi}{7}\right) + \tan\left(\frac{2\pi}{7}\right) - \tan\left(\frac{2\pi}{7}\right) - \tan\left(\frac{\pi}{7}\right)$$

$$= 0$$

$$\boxed{}$$

إذن

$$= 2 \times 1 = 2$$

$$C = 2$$

إذن

تمرين 15:

نعتبر التعبيرات التالية :

$$A = \cos\frac{\pi}{5} + \cos\frac{2\pi}{5} + \cos\frac{3\pi}{5} + \cos\frac{4\pi}{5}$$

$$B = \sin\left(\frac{11\pi}{26}\right) + \sin\left(\frac{3\pi}{26}\right) + \cos\left(\frac{12\pi}{13}\right) + \cos\left(\frac{8\pi}{13}\right)$$

$$C = \sin\left(\frac{\pi}{14}\right) + \sin\left(\frac{3\pi}{14}\right) + \cos\left(\frac{5\pi}{14}\right) + \cos\left(\frac{4\pi}{7}\right) + \cos\left(\frac{5\pi}{7}\right) + \cos\left(\frac{6\pi}{7}\right)$$

. احسب A و B و C

الجواب :

لدينا :

$$A = \cos\frac{\pi}{5} + \cos\frac{2\pi}{5} + \cos\frac{3\pi}{5} + \cos\frac{4\pi}{5}$$

$$= \cos\frac{\pi}{5} + \cos\frac{2\pi}{5} + \cos\left(\pi - \frac{2\pi}{14}\right) + \cos\left(\pi - \frac{\pi}{5}\right)$$

$$= \cos\left(\frac{\pi}{5}\right) + \cos\left(\frac{2\pi}{5}\right) - \cos\left(\frac{2\pi}{5}\right) - \cos\left(\frac{\pi}{5}\right)$$

$$= 0$$

$$A = 0$$

إذن

$$B = \sin\left(\frac{11\pi}{26}\right) + \sin\left(\frac{3\pi}{26}\right) + \cos\left(\frac{12\pi}{13}\right) + \cos\left(\frac{8\pi}{13}\right)$$

لدينا :

$$= \sin\left(\frac{13\pi - 2\pi}{26}\right) + \sin\left(\frac{13\pi - 10\pi}{26}\right) + \cos\left(\pi - \frac{\pi}{13}\right) + \cos\left(\pi - \frac{5\pi}{13}\right)$$

$$= \sin\left(\frac{\pi}{2} - \frac{\pi}{13}\right) + \sin\left(\frac{\pi}{2} - \frac{5\pi}{13}\right) + \cos\left(\pi - \frac{\pi}{13}\right) + \cos\left(\pi - \frac{5\pi}{13}\right)$$

$$= \cos\frac{\pi}{13} + \cos\frac{5\pi}{13} - \cos\frac{\pi}{13} - \cos\frac{5\pi}{13}$$

$$= 0$$

$$B = 0$$

إذن

$$C = \sin\left(\frac{\pi}{14}\right) + \sin\left(\frac{3\pi}{14}\right) + \sin\left(\frac{5\pi}{14}\right) + \cos\left(\frac{8\pi}{14}\right) + \cos\left(\frac{10\pi}{14}\right) + \cos\left(\frac{12\pi}{14}\right)$$

$$= \sin\left(\frac{\pi}{14}\right) + \sin\left(\frac{3\pi}{14}\right) + \sin\left(\frac{5\pi}{14}\right) + \cos\left(\frac{7\pi + \pi}{14}\right) + \cos\left(\frac{7\pi + 3\pi}{14}\right) + \cos\left(\frac{7\pi + 5\pi}{14}\right)$$

$$= \sin\left(\frac{\pi}{14}\right) + \sin\left(\frac{3\pi}{14}\right) + \sin\left(\frac{5\pi}{14}\right) + \cos\left(\frac{\pi}{2} + \frac{\pi}{14}\right) + \cos\left(\frac{\pi}{2} + \frac{3\pi}{14}\right) + \cos\left(\frac{\pi}{2} + \frac{5\pi}{14}\right)$$

$$= \sin\cancel{\left(\frac{\pi}{14}\right)} + \sin\cancel{\left(\frac{3\pi}{14}\right)} + \sin\cancel{\left(\frac{5\pi}{14}\right)} - \sin\cancel{\left(\frac{\pi}{14}\right)} - \sin\cancel{\left(\frac{3\pi}{14}\right)} - \sin\cancel{\left(\frac{5\pi}{14}\right)}$$



$$= 0 \quad C = 0 \quad \text{إذن}$$

تمرين 16:

حسب مايلي :

$$A = \sin^2 \frac{\pi}{8} + \sin^2 \frac{3\pi}{8} + \sin^2 \frac{5\pi}{8} + \sin^2 \frac{7\pi}{8}$$

$$B = \sin^2 \frac{\pi}{7} + \sin^2 \frac{\pi}{9} + \cos^2 \frac{6\pi}{7} + \cos^2 \frac{8\pi}{9}$$

الجواب :

$$A = \sin^2 \frac{\pi}{8} + \sin^2 \frac{3\pi}{8} + \sin^2 \frac{5\pi}{8} + \sin^2 \frac{7\pi}{8}$$

$$= \sin^2 \left(\frac{\pi}{8} \right) + \sin^2 \left(\frac{\pi}{2} - \frac{\pi}{8} \right) + \sin^2 \left(\frac{\pi}{2} + \frac{\pi}{8} \right) + \sin^2 \left(\pi - \frac{\pi}{8} \right)$$

$$= \sin^2 \left(\frac{\pi}{8} \right) + \cos^2 \left(\frac{\pi}{8} \right) + \left(-\cos \frac{\pi}{8} \right)^2 + \sin^2 \left(\frac{\pi}{8} \right)$$

$$= \sin^2 \left(\frac{\pi}{8} \right) + \cos^2 \left(\frac{\pi}{8} \right) + \cos^2 \frac{\pi}{8} + \sin^2 \frac{\pi}{8}$$

$$= 2 \left(\sin^2 \frac{\pi}{8} + \cos^2 \frac{\pi}{8} \right)$$

$$= 2 \times 1 = 2$$

$$A = 2$$

إذن

$$B = \sin^2 \frac{\pi}{7} + \sin^2 \frac{\pi}{9} + \cos^2 \frac{6\pi}{7} + \cos^2 \frac{8\pi}{9}$$

$$= \sin^2 \frac{\pi}{7} + \sin^2 \frac{\pi}{9} + \cos^2 \left(\pi - \frac{\pi}{7} \right) + \cos^2 \left(\pi - \frac{\pi}{9} \right)$$

$$= \sin^2 \frac{\pi}{7} + \sin^2 \frac{\pi}{9} + \left(-\cos \frac{\pi}{8} \right)^2 + \left(-\cos \frac{\pi}{9} \right)^2$$

$$= \sin^2 \left(\frac{\pi}{7} \right) + \sin^2 \left(\frac{\pi}{9} \right) + \cos^2 \left(\frac{\pi}{7} \right) + \cos^2 \left(\frac{\pi}{9} \right)$$

$$= \left(\sin^2 \left(\frac{\pi}{7} \right) + \cos^2 \left(\frac{\pi}{7} \right) \right) + \left(\sin^2 \left(\frac{\pi}{9} \right) + \cos^2 \left(\frac{\pi}{9} \right) \right)$$

$$= 1 + 1 = 2$$

$$B = 2$$

$$\cos\left(-\frac{\pi}{8}\right) = \cos\left(\frac{\pi}{8}\right) \quad \text{لدينا} \quad (2)$$

$$= \frac{\sqrt{2 + \sqrt{2}}}{2}$$

$$\boxed{\cos\left(-\frac{\pi}{8}\right) = \frac{\sqrt{2 + \sqrt{2}}}{2}}$$

إذن

$$\sin\left(-\frac{7\pi}{8}\right) = -\sin\left(\frac{7\pi}{8}\right) = -\sin\left(\pi - \frac{\pi}{8}\right)$$

$$= -\sin\left(\frac{\pi}{8}\right)$$

$$= -\frac{\sqrt{2 - \sqrt{2}}}{2}$$

$$\boxed{\sin\left(-\frac{7\pi}{8}\right) = -\frac{\sqrt{2 - \sqrt{2}}}{2}}$$

إذن

$$\cos\left(\frac{513\pi}{8}\right)$$

- (3)

$$= \cos\left(\frac{512\pi + \pi}{8}\right)$$

$$= \cos\left(64\pi + \frac{\pi}{8}\right)$$

$$= \cos\left(\frac{\pi}{8}\right)$$

$$= \frac{\sqrt{2 + \sqrt{2}}}{2}$$

$$\cos\left(\frac{513\pi}{8}\right) = \frac{\sqrt{2 + \sqrt{2}}}{2}$$

إذن

$$\sin\left(\frac{37\pi}{8}\right)$$

لدينا

$$= \sin\left(\frac{40\pi - 3\pi}{8}\right)$$

$$= \sin\left(5\pi - \frac{3\pi}{8}\right)$$

$$= \sin\left(\pi - \frac{3\pi}{8}\right)$$

تمرين 17:

$$\cos\left(\frac{\pi}{8}\right) = \frac{\sqrt{2 + \sqrt{2}}}{2} : \text{ نعلم أن}$$

$$\cdot \cos\frac{3\pi}{8}, \sin\frac{\pi}{8} - (1)$$

$$\sin\left(-\frac{7\pi}{8}\right), \sin\left(-\frac{\pi}{8}\right) : \text{ استنتج} - (2)$$

$$\sin\left(\frac{37\pi}{8}\right) \text{ و } \cos\left(\frac{513\pi}{8}\right) : \text{ احسب} - (3)$$

$$\tan\left(\frac{25\pi}{8}\right) \text{ و}$$

الجواب :

$$\cos^2\left(\frac{\pi}{8}\right) + \sin^2\left(\frac{\pi}{8}\right) = 1 : \text{ نعلم أن} - (1)$$

$$\left(\frac{\sqrt{2 + \sqrt{2}}}{2}\right)^2 + \sin^2\left(\frac{\pi}{8}\right) = 1 \quad \text{أي أن}$$

$$\sin^2\left(\frac{\pi}{8}\right) = 1 - \frac{2 + \sqrt{2}}{4}$$

$$= \frac{4 - 2 - \sqrt{2}}{4}$$

$$= \frac{2 - \sqrt{2}}{4}$$

$$\sin\frac{\pi}{8} = \frac{\sqrt{2 - \sqrt{2}}}{2} \quad \text{أو} \quad \sin\frac{\pi}{8} = -\frac{\sqrt{2 - \sqrt{2}}}{2}$$

$$\boxed{\sin\left(\frac{\pi}{8}\right) = \frac{\sqrt{2 - \sqrt{2}}}{2}} \quad \text{بما أن } 0 < \frac{\pi}{8} < \frac{\pi}{2}$$

$$\cos\left(\frac{3\pi}{8}\right) = \cos\left(\frac{4\pi - \pi}{8}\right)$$

$$= \cos\left(\frac{\pi}{2} - \frac{\pi}{8}\right)$$

$$= \sin\frac{\pi}{8}$$

$$= \frac{\sqrt{2 - \sqrt{2}}}{2}$$

$$\boxed{\cos\left(\frac{3\pi}{8}\right) = \frac{\sqrt{2 - \sqrt{2}}}{2}}$$

إذن

تمرين 17

ليكن $x \neq \frac{\pi}{2}$ و $[0, \pi]$ من المجال
نضع

$$P(x) = 2\left[1 - \cos^2\left(\frac{\pi}{2} + x\right)\right] - \cos(\pi - x) \sin(\pi + x)$$

$\cdot P\left(\frac{\pi}{4}\right)$ و $P(0)$ ثم احسب $P(x)$ بسط - 1

$$P(x) = \frac{2 - \tan x}{1 + \tan^2 x} \quad - 1 - أ - بین أن :$$

$$B - احسب P(x) و P\left(\frac{\pi}{3}\right)$$

$$x \in \left[0, \frac{\pi}{2}\right] \quad - 3 - أدا علمت أن 2 = P(x) و$$

فاحسب x واستنتج

الجواب : - 1

$$P(x) = 2\left[1 - \cos^2\left(\frac{\pi}{2} + x\right)\right] - \cos(\pi - x) \sin(\pi + x)$$

$$= 2\left[1 - (-\sin x)^2\right] - (-\cos x)(-\sin x)$$

$$= 2(1 - \sin^2 x) - (-\cos x) \times (-\sin x)$$

$$= 2\cos^2 x - \cos x \cdot \sin x$$

$$P(x) = 2\cos^2 x - \cos x \cdot \sin x \quad \text{إذن}$$

$$P(0) = 2\cos^2 0 - (\cos 0) \times (\sin 0) \quad \text{لدينا}$$

$$= 2 \times 1 - 1 \times 0$$

$$= 2$$

$$P(0) = 2 \quad \text{إذن}$$

$$P\left(\frac{\pi}{4}\right) = 2\cos^2 \frac{\pi}{4} - \cos\left(\frac{\pi}{4}\right) \times \sin\left(\frac{\pi}{4}\right)$$

$$= \sin\left(\frac{3\pi}{8}\right)$$

$$= \sin\left(\frac{3\pi}{8}\right)$$

$$= \sin\left(\frac{\pi}{2} - \frac{\pi}{8}\right)$$

$$= \cos\frac{\pi}{8}$$

$$\sin\left(\frac{37\pi}{8}\right) = \frac{\sqrt{2 + \sqrt{2}}}{2} \quad \text{إذن}$$

لدينا

$$\tan\left(\frac{25\pi}{8}\right) = \tan\left(\frac{24\pi + \pi}{8}\right)$$

$$= \tan\left(3\pi + \frac{\pi}{8}\right)$$

$$= \tan\frac{\pi}{8}$$

$$= \frac{\sqrt{2 - \sqrt{2}}}{2}$$

$$= \frac{2}{\sqrt{2 + \sqrt{2}}}$$

$$= \frac{\sqrt{2 - \sqrt{2}} \cdot \sqrt{2 - \sqrt{2}}}{\sqrt{4 - 2}}$$

$$= \frac{2 - \sqrt{2}}{\sqrt{2}}$$

$$= \frac{2}{\sqrt{2}} - 1$$

$$= \sqrt{2} - 1$$

$$\tan\frac{\pi}{8} = \sqrt{2} - 1 \quad \text{إذن}$$

$$\frac{2 - \tan x}{1 + \tan^2 x} = 2 \quad \text{لدينا} \quad P(x) = 2 \quad \text{تكافى}$$

$$2 - \tan x = 2 + 2\tan^2 x \quad \text{تكافى}$$

$$- \tan x = 2\tan^2 x$$

$$2\tan^2 x + \tan x = 0 \quad \text{أي أن}$$

$$\tan x (2\tan x + 1) = 0$$

$$\tan x = 0 \quad \text{أي أن} \quad \tan x = -\frac{1}{2}$$

$$\tan x \geq 0 : \quad \text{فإن } x \in \left[0, \frac{\pi}{2}\right] \quad \text{و بما أن}$$

$$\tan x = 0 \quad \text{إذن}$$

$$x \in \left[0, \frac{\pi}{2}\right] \quad \text{و بما أن}$$

$$x = 0 \quad \text{فإن}$$

تمرين 18:

ليكن x من المجال $[\pi; \pi + \pi]$ و $x \neq \frac{\pi}{2}$

$$x \neq -\frac{\pi}{2}$$

$$P(x) = \frac{1 - \sin x}{1 + \sin x} + \frac{1 + \sin x}{1 - \sin x} \quad \text{و}$$

$$P(x) = 2(1 + 2\tan^2 x) \quad \text{أي أن} - 1$$

$$P\left(\frac{\pi}{6}\right) \text{ و } P\left(\frac{\pi}{3}\right) \quad \text{أي احسب} - 2$$

$$P\left(-\frac{\pi}{3}\right) \text{ و } P\left(\frac{5\pi}{6}\right) \text{ و } P\left(\frac{2\pi}{3}\right) \quad \text{ب - استسق}$$

$$P\left(-\frac{\pi}{6}\right) \quad \text{و}$$

$$P(x) = 14 \quad \text{إذا علمت أن :} - 3$$

$$\sin x \text{ ثم } \cos x \text{ و } \tan x \quad \text{فاحسب } 0 < x < \frac{\pi}{2} \quad \text{و}$$

$$= 2\left(\frac{\sqrt{2}}{2}\right)^2 - \frac{\sqrt{2}}{2} \times \frac{\sqrt{2}}{2}$$

$$= 1 - \frac{1}{2}$$

$$= \frac{1}{2} \quad P\left(\frac{\pi}{4}\right) = \frac{1}{2}$$

- 1 - 2

$$P(x) = 2\cos^2 x - \cos x \cdot \sin x \quad \text{لدينا}$$

$$= 2\cos^2 x - \tan x \cdot \cos x \cdot \cos x$$

$$= 2\cos^2 x - (\tan x) \cdot \cos^2 x$$

$$= \cos^2 x \cdot (2 - \tan x)$$

$$= \frac{1}{1 + \tan^2 x} (2 - \tan x)$$

$$P(x) = \frac{2 - \tan x}{1 + \tan^2 x}$$

إذن

$$P\left(\frac{\pi}{3}\right) = \frac{2 - \tan \frac{\pi}{3}}{2 + \tan^2 \frac{\pi}{3}} \quad \text{لدينا} - \text{ب}$$

$$= \frac{2 - \sqrt{3}}{1 + (\sqrt{3})^2}$$

$$P\left(\frac{\pi}{3}\right) = \frac{2 - \sqrt{3}}{4}$$

$$P(\pi) = \frac{2 - \tan \pi}{1 + \tan^2 \pi}$$

$$= \frac{2 - 0}{1 + 0^2}$$

$$= 2$$

$$P(\pi) = 2$$

إذن

$$P(x) = 2 ; x \in \left[0, \frac{\pi}{2}\right] - 3$$

$$= 2\left(1 + \frac{2}{3}\right)$$

$$= \frac{10}{3}$$

$$P\left(\frac{\pi}{6}\right) = \frac{10}{3}$$

$$P(x) = P(\pi - x) = P(-x) \quad \text{لدينا} - \text{لدينا}$$

$$\begin{aligned} P\left(\frac{2\pi}{3}\right) &= P\left(\pi - \frac{\pi}{3}\right) \\ &= P\left(\frac{\pi}{3}\right) \end{aligned}$$

$$P\left(\frac{2\pi}{3}\right) = 14$$

$$\begin{aligned} P\left(\frac{5\pi}{6}\right) &= P\left(\pi - \frac{\pi}{6}\right) \\ &= P\left(\frac{\pi}{6}\right) \end{aligned}$$

$$P\left(\frac{5\pi}{6}\right) = \frac{10}{3}$$

$$\begin{aligned} P\left(-\frac{\pi}{3}\right) &= P\left(\frac{\pi}{3}\right) \\ &= 14 \end{aligned}$$

$$P\left(-\frac{\pi}{6}\right) = P\left(\frac{\pi}{6}\right) = \frac{10}{3}$$

$$P(x) = 14$$

$$2(1 + 2\tan^2 x) = 14$$

$$1 + 2\tan^2 x = 7$$

$$2\tan^2 x = 6$$

$$\tan^2 x = 3$$

$$\tan x = \sqrt{3} \quad \text{أو} \quad \tan x = -\sqrt{3}$$

$$\tan x > 0 \quad \text{فإن} \quad 0 < x < \frac{\pi}{2} \quad \text{عما} \quad \text{أن}$$

الجواب :

$$P(x) = \frac{1 - \sin x}{1 + \sin x} + \frac{1 + \sin x}{1 - \sin x} : \text{لدينا} - 1$$

$$= \frac{(1 - \sin x)^2 + (1 + \sin x)^2}{(1 + \sin x)(1 - \sin x)}$$

$$= \frac{1 + \sin^2 x - 2\sin x + 1 + \sin^2 x + 2\sin x}{1 - \sin^2 x}$$

$$= \frac{2 + 2\sin^2 x}{\cos^2 x}$$

$$= \frac{2 + 2(1 - \cos^2 x)}{\cos^2 x}$$

$$= \frac{4 - 2\cos^2 x}{\cos^2 x}$$

$$\begin{aligned} &= \frac{2\left(2 - \frac{1}{1 + \tan^2 x}\right)}{\frac{1}{1 + \tan^2 x}} \\ &= \frac{2\left(\frac{2 + 2\tan^2 x - 1}{1 + \tan^2 x}\right)}{\frac{1}{1 + \tan^2 x}} \end{aligned}$$

$$P(x) = 2(1 + 2\tan^2 x)$$

$$\begin{aligned} P\left(\frac{\pi}{3}\right) &= 2\left(1 + 2\tan^2 \frac{\pi}{3}\right) \quad - 1 - 2 \\ &= 2(1 + 2 \cdot (\sqrt{3})^2) \\ &= 2(1 + 6) \end{aligned}$$

$$P\left(\frac{\pi}{3}\right) = 14$$

$$\begin{aligned} P\left(\frac{\pi}{6}\right) &= 2\left(1 + 2\tan^2 \frac{\pi}{6}\right) \\ &= 2\left(1 + 2 \times \left(\frac{\sqrt{3}}{3}\right)^2\right) \end{aligned}$$

الجواب :

لدينا

$$A(x) = \cos^2 x + 3 \sin x \cdot \cos x - 2 \sin^2 x$$

$$= \cos^2 x \left(1 + \frac{3 \sin x \cdot \cos x}{\cos^2 x} - \frac{2 \sin^2 x}{\cos^2 x} \right)$$

$$= \cos^2 x \left(1 + \frac{3 \sin x}{\cos x} - 2 \cdot \left(\frac{\sin x}{\cos x} \right)^2 \right)$$

$$A(x) = \cos^2 x \left(1 + 3 \tan x - 2 \tan^2 x \right) \quad \text{إذن}$$

$$\cos x = \frac{\sqrt{5}}{5} \quad \text{لدينا} \quad - 2$$

$$\cos^2 x = \frac{1}{5} \quad \text{إذن}$$

$$\frac{1}{\cos^2 x} = 5 \quad \text{ومنه}$$

$$1 + \tan^2 x = 5 \quad \text{أي أن}$$

$$\tan^2 x = 4 \quad \text{إذن}$$

$$\tan x = -2 \quad \text{أو} \quad \tan x = 2 \quad \text{ومنه}$$

$$0 < x < \frac{\pi}{2} \quad \text{ويعاً أن}$$

$$\tan x > 0 \quad \text{فإن}$$

$$\tan x = 2 \quad \text{إذن}$$

ومنه

$$A(x) = \frac{1}{5} (1 + 3 \times 2 - 2 \times 4)$$

$$= \frac{1}{5} (1 + 6 - 8)$$

$$= \frac{1}{5}$$

$$A(x) = \frac{1}{5}$$

إذن

$$\tan x = \sqrt{3} \quad \text{ومنه}$$

$$\cos^2 x = \frac{1}{1 + \tan^2 x} \quad \text{لدينا}$$

$$= \frac{1}{1 + (\sqrt{3})^2}$$

$$= \frac{1}{4} \quad \cos x = \sqrt{\frac{1}{4}} \quad \text{أو} \quad \cos x = -\sqrt{\frac{1}{4}}$$

$$\cos x = \frac{1}{2} \quad \text{أو} \quad \cos x = -\frac{1}{2}$$

$$\cos x = \frac{1}{2} \quad \text{وعما أَن} \quad \cos x > 0 \quad \text{فإن}$$

$$\tan x = \frac{\sin x}{\cos x} \quad \text{لدينا}$$

$$\sin x = \tan x \cos x$$

$$= \sqrt{3} \times \frac{1}{2}$$

$$= \frac{\sqrt{3}}{2}$$

$$\sin x = \frac{\sqrt{3}}{2} \quad \text{إذن}$$

تمرين 19:

ليكن x عدداً حقيقياً بحيث : $x \neq \frac{\pi}{2} + k\pi$; $k \in \mathbf{Z}$;

$$A(x) = \cos^2 x + 3 \sin x \cdot \cos x - 2 \sin^2 x$$

- بين أن

$$A(x) = \cos^2 x (1 + 3 \tan x - 2 \tan^2 x)$$

- احسب $A(x)$ إذا علمت أن :

$$0 < x < \frac{\pi}{2} \quad \text{و} \quad \cos x = \frac{\sqrt{5}}{5}$$

$$A\left(-\frac{\pi}{2}\right) = \sin\left(-\frac{\pi}{2}\right) - \cos\left(-\frac{\pi}{2}\right) \quad \text{لدينا}$$

$$= -\sin\frac{\pi}{2} - \cos\frac{\pi}{2}$$

$$= -1 - 0$$

$$A\left(-\frac{\pi}{2}\right) = -1 \quad \text{إذن}$$

$$A\left(\frac{13\pi}{3}\right) = \sin\left(\frac{13\pi}{3}\right) - \cos\left(\frac{13\pi}{3}\right) \quad \text{لدينا}$$

$$= \sin\left(4\pi + \frac{\pi}{3}\right) - \cos\left(4\pi + \frac{\pi}{3}\right)$$

$$= \sin\left(\frac{\pi}{3}\right) - \cos\left(\frac{\pi}{3}\right)$$

$$= \frac{\sqrt{3}}{2} - \frac{1}{2}$$

$$= \frac{\sqrt{3} - 1}{2}$$

$$A\left(\frac{13\pi}{3}\right) = \frac{\sqrt{3} - 1}{2} \quad \text{إذن}$$

$$A(x) = \sin x - \cos x \quad \text{لدينا}$$

$$[A(x)]^2 = (\sin x - \cos x)^2 \quad \text{إذن}$$

$$= \sin^2 x - 2\sin x \cdot \cos x + \cos^2 x$$

$$= 1 - 2\sin x \cdot \cos x$$

$$= 1 - 2\tan x \cdot \cos x \cdot \cos x$$

$$= 1 - 2 \cdot \tan x \cdot \cos^2 x$$

$$= 1 - 2\tan x \times \frac{1}{1 + \tan^2 x}$$

$$= 1 - \frac{2\tan x}{1 + \tan^2 x}$$

$$= \frac{1 + \tan^2 x - 2\tan x}{1 + \tan^2 x}$$

$$[A(x)]^2 = \frac{(1 - \tan x)^2}{1 + \tan^2 x} \quad \text{إذن}$$

تمرين 20:

ليكن x من \mathbb{R} . نضع

$$A(x) = 3\cos(3\pi + x) - 2\sin(\pi + x)$$

$$- \cos\left(\frac{9\pi}{2} + x\right) + 2\sin\left(\frac{9\pi}{2} - x\right)$$

. $\cos x$ بدلالة $\sin x$ - 1

$$\cdot A\left(\frac{13\pi}{3}\right) \text{ و } A\left(\frac{-\pi}{2}\right) \cdot A(0) - 2$$

. $A(x)$ بدلالة $\tan x$ لكل x - 3

$$\cdot k \in \mathbb{Z}, \frac{\pi}{2} + k\pi \text{ يخالف}$$

الجواب :

- 1 - لدina

$$A(x) = 3\cos(3\pi + x) - 2\sin(\pi + x) +$$

$$\cos\left(\frac{9\pi}{2} + x\right) + 2\sin\left(\frac{9\pi}{2} - x\right)$$

$$= 3\cos(2\pi + \pi + x) + 2\sin x +$$

$$\cos\left(\frac{4\pi + \pi}{2} + x\right) + 2\sin\left(\frac{8\pi + \pi}{2} - x\right)$$

$$= -3\cos x + 2\sin x + \cos\left(\frac{\pi}{2} + x\right) +$$

$$2\sin\left(\frac{\pi}{2} - x\right)$$

$$= -3\cos x + 2\sin x - \sin x + 2\cos x$$

$$= \sin x - \cos x$$

$$A(x) = \sin x - \cos x \quad \text{إذن}$$

$$A(0) = \sin 0 - \cos 0 \quad \text{لدينا} - 2$$

$$A(0) = -1 \quad \text{إذن}$$



$$\begin{aligned}
 &= -2 + 3\cos^2 x + 3\cos^2 x \times \tan x \\
 &= -2 + 3 \cos^2 x(1 + \tan x) \\
 &= -2 + 3 \times \frac{1}{1 + \tan^2 x} (1 + \tan x)
 \end{aligned}$$

$$E(x) = -2 + 3 \times \left(\frac{1 + \tan x}{1 + \tan^2 x} \right)$$

إذن

$$E(x) = 1 \quad \text{لدينا}$$

$$-2 + 3 \frac{1 + \tan x}{1 + \tan^2 x} = 1 \quad \text{نكافى}$$

$$3 \left(\frac{1 + \tan x}{1 + \tan^2 x} \right) = 3 \quad \text{أى أن}$$

$$\frac{1 + \tan x}{1 + \tan^2 x} = 1 \quad \text{ومنه}$$

$$1 + \tan x = 1 + \tan^2 x \quad \text{إذن}$$

$$\tan x = \tan^2 x \quad \text{أى أن}$$

$$\tan x - \tan^2 x = 0 \quad \text{أى أن}$$

$$\tan x (1 - \tan x) = 0 \quad \text{أى أن}$$

$$\tan x = 0 \quad \text{أى} \quad \text{أو} \quad \tan x = 1 \quad \text{إذن}$$

ويعاً أن $0 \neq x \neq \pi$ فـان $x \neq 0$ و $\tan x \neq 0$

$$\tan x = 1 \quad \text{إذن}$$

$$\cos^2 x = \frac{1}{1 + \tan^2 x} \quad \text{لدينا}$$

$$= \frac{1}{1 + 1}$$

$$\cos^2 x = \frac{1}{2}$$

$$\cos x = \frac{\sqrt{2}}{2} \quad \text{أى} \quad \cos x = -\frac{\sqrt{2}}{2}$$

$$\cos x = \frac{\sqrt{2}}{2} \quad \text{فـان} \quad 0 < x < \frac{\pi}{2} \quad \text{وعـاً أن}$$

تمرين 21:

ليـكـن x مـن $[0; \pi]$. نـصـع

$$E(x) = \cos^2 x + 3\cos x \cdot \sin x - 2\sin^2 x$$

1 - اـحـسـب $E(\pi)$ و $E(0)$

2 - ليـكـن x مـن المـجاـل $[0, \frac{\pi}{2}]$

$$E(x) = -2 + 3 \left(\frac{1 + \tan x}{1 + \tan^2 x} \right)$$

3 - إـذـا عـلـمـتـ أـن $E(x) = 1$

فـاحـسـب $\cos x$ ثـم $\tan x$

الجواب :

1 - لـدـيـنـا

$$E(x) = \cos^2 x + 3\cos x \cdot \sin x - 2\sin^2 x$$

$$E(0) = \cos^2 0 + 3\cos 0 \cdot \sin 0 - 2\sin^2 0$$

$$= 1 + 3 \times 0 \times 1 - 2 \times 0^2 \quad \text{إـذـن}$$

$$E(0) = 1$$

$$E(\pi) = \cos^2 \pi + 3(\cos \pi) \times (\sin \pi) - 2\sin^2 \pi$$

$$= (-1)^2 + 3(-1)0 - 2 \cdot 0^2$$

$$= 1$$

$$E(\pi) = 1$$

إـذـن

2 - لـدـيـنـا

$$E(x) = \cos^2 x + 3\cos x \cdot \sin x - 2\sin^2 x$$

$$= \cos^2 x + 3\cos x \times \tan x \cdot \cos x - 2(1 - \cos^2 x)$$

$$= \cos^2 x + 3\cos^2 x \times \tan x - 2 + 2\cos^2 x$$



تمرين 22:

إذن

$$A(x) = -(\cos x + \sin x) \times \sin x \cdot \cos x$$

2 - لدينا

$$\begin{aligned} A(0) &= -(\sin 0 + \cos 0) \times \sin 0 \times \cos 0 \\ &= -1 \times 0 \times 1 \\ &= 0 \end{aligned}$$

$$A(0) = 0$$

إذن

لدينا

$$\begin{aligned} A\left(\frac{\pi}{4}\right) &= -\left(\sin \frac{\pi}{4} + \cos \frac{\pi}{4}\right) \times \sin \frac{\pi}{4} \cdot \cos \frac{\pi}{4} \\ &= -\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\right) \times \frac{\sqrt{2}}{2} \times \frac{\sqrt{2}}{2} \\ &= -\sqrt{2} \times \frac{2}{4} \\ &= -\frac{\sqrt{2}}{2} \end{aligned}$$

$$A\left(\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$$

إذن

لدينا

$$\begin{aligned} A\left(\frac{\pi}{3}\right) &= -\left(\sin \frac{\pi}{3} + \cos \frac{\pi}{3}\right) \times \sin \frac{\pi}{3} \cdot \cos \frac{\pi}{3} \\ &= -\left(\frac{\sqrt{3}}{2} + \frac{1}{2}\right) \times \frac{\sqrt{3}}{2} \times \frac{1}{2} \\ &= -\left(\frac{\sqrt{3} + 1}{2}\right) \times \frac{\sqrt{3}}{4} \end{aligned}$$

$$A\left(\frac{\pi}{3}\right) = \frac{-(3 + \sqrt{3})}{8}$$

إذن

لدينا - ١ - ٣

$$\begin{aligned} A\left(\frac{\pi}{2} - x\right) &= -\left[\sin\left(\frac{\pi}{2} - x\right) + \cos\left(\frac{\pi}{2} - x\right)\right] \cdot \\ &\quad \sin\left(\frac{\pi}{2} - x\right) \times \cos\left(\frac{\pi}{2} - x\right) \\ &= -(\cos x + \sin x) \cos x \sin x \\ &= A(x) \end{aligned}$$

ليكن x من \mathbb{R} . نضع

$$A(x) = \cos^3 x + \sin^3 x + \cos(7\pi + x) - \sin(x - 9\pi)$$

1 - بين أن :

$$A(x) = -(\sin(x) + \cos(x)) \cdot \sin(x) \cdot \cos(x)$$

2 - احسب :

$$A\left(\frac{\pi}{3}\right) \text{ و } A\left(\frac{\pi}{4}\right) \cdot A(0)$$

3 - بين أن :

$$A\left(\frac{\pi}{2} - x\right) = A(x)$$

ب - استنتج حساب : $A\left(\frac{\pi}{6}\right) \text{ و } A\left(\frac{\pi}{2}\right)$

الجواب :

1 - لدينا :

$$A(x) = \cos^3 x + \sin^3 x + \cos(6\pi + \pi + x) + \sin(x - \pi - 8\pi)$$

$$= \cos^3 x + \sin^3 x + \cos(\pi + x) + \sin(x - \pi)$$

$$= (\cos x + \sin x) \times (\cos^2 x - \sin x \cos x +$$

$$\sin^2 x) - \cos x - \sin x$$

$$= (\cos x + \sin x) \times (1 - \sin x \cos x) -$$

$$(\cos x + \sin x)$$

$$= (\cos x + \sin x) \times (1 - \sin x \cdot \cos x - 1)$$

$$= (\cos x + \sin x) (-\sin x \cdot \cos x)$$

$$\sin^2\left(\frac{\pi}{5}\right) = 1 - \frac{6+2\sqrt{5}}{16}$$

- ا

$$\sin^2\frac{\pi}{5} = \frac{\sqrt{10-2\sqrt{5}}}{16}$$

$$\sin\frac{\pi}{5} = \frac{\sqrt{10-2\sqrt{5}}}{4}$$

$$\sin\frac{\pi}{5} = -\frac{\sqrt{10-2\sqrt{5}}}{4}$$

أو

$$0 < \frac{\pi}{5} < \frac{\pi}{2}$$

وعما أن

$$\sin\frac{\pi}{5} > 0$$

فإن

$$\boxed{\sin\left(\frac{\pi}{5}\right) = \frac{\sqrt{10-2\sqrt{5}}}{4}}$$

ومنه

$$\cos\frac{4\pi}{5} = \cos\left(\frac{5\pi-\pi}{5}\right)$$

لدينا

$$= \cos\left(\pi - \frac{\pi}{5}\right)$$

$$= -\cos\frac{\pi}{5}$$

$$= -\frac{\sqrt{5}+1}{4}$$

$$\sin\frac{4\pi}{5} = \sin\left(\frac{5\pi-\pi}{5}\right)$$

لدينا

$$= \sin\left(\pi - \frac{\pi}{5}\right)$$

$$= +\sin\frac{\pi}{5}$$

$$= +\frac{\sqrt{10-2\sqrt{5}}}{4}$$

$$\boxed{\sin\frac{4\pi}{5} = +\frac{\sqrt{10-2\sqrt{5}}}{4}}$$

$$\cos\left(\frac{7\pi}{10}\right) = \cos\left(\frac{5\pi}{10} + \frac{2\pi}{10}\right)$$

- 2

$$A\left(\frac{\pi}{2} - x\right) = A(x)$$

إذن

$$\frac{\pi}{2} - 0 = \frac{\pi}{2}$$

$$A\left(\frac{\pi}{2} - 0\right) = A\left(\frac{\pi}{2}\right)$$

$$A(0) = A\left(\frac{\pi}{2}\right)$$

$$A\left(\frac{\pi}{2}\right) = 0$$

$$\frac{\pi}{6} - \frac{\pi}{2} - \frac{\pi}{3}$$

$$A\left(\frac{\pi}{6}\right) = A\left(\frac{\pi}{2} - \frac{\pi}{3}\right)$$

$$A\left(\frac{\pi}{6}\right) = A\left(\frac{\pi}{3}\right)$$

$$A\left(\frac{\pi}{3}\right) = \frac{-(3+\sqrt{3})}{8}$$

$$A\left(\frac{\pi}{6}\right) = \frac{-(3+\sqrt{3})}{8}$$

تمرين 23:

$$\text{علمـاً أـن : } \cos\left(\frac{\pi}{5}\right) = \frac{\sqrt{5}+1}{4}$$

$$\sin\left(\frac{4\pi}{5}\right), \cos\frac{4\pi}{5}, \sin\frac{\pi}{5} - 1 \quad \text{- أـحسب}$$

$$\tan\frac{3\pi}{10}, \sin\frac{-3\pi}{10}, \cos\frac{7\pi}{10} - 2 \quad \text{- أـحسب}$$

$$\cos\frac{101\pi}{10}; \sin\frac{-84\pi}{10} - 3$$

الجواب :

1 - نعلمـاً أـن

$$\cos^2\left(\frac{\pi}{5}\right) + \sin^2\left(\frac{\pi}{5}\right) = 1$$

$$\left(\frac{\sqrt{5}+1}{4}\right)^2 + \sin^2\left(\frac{\pi}{5}\right) = 1$$

تـكـافـي

$$= \frac{(\sqrt{5} + 5)\sqrt{2} \cdot \sqrt{5 + \sqrt{5}}}{20}$$

$$= \frac{\sqrt{2} \times (\sqrt{5 + \sqrt{5}})}{20}$$

لدينا - 3

$$\sin\left(-\frac{84\pi}{5}\right) = \sin\left(\frac{-85\pi + \pi}{5}\right)$$

$$= \sin\left(-17\pi + \frac{\pi}{5}\right)$$

$$= \sin\left(-18\pi + \pi + \frac{\pi}{5}\right)$$

$$= \sin\left(\pi + \frac{\pi}{5}\right)$$

$$= -\sin\left(\frac{\pi}{5}\right)$$

$$\sin\left(-\frac{84\pi}{5}\right) = \frac{\sqrt{10-2\sqrt{5}}}{4}$$

إذن

لدينا

$$\cos\left(\frac{101\pi}{5}\right) = \cos\left(\frac{100\pi + \pi}{5}\right)$$

$$= \cos\left(20\pi + \frac{\pi}{5}\right)$$

$$= \cos\frac{\pi}{5}$$

$$\cos\left(\frac{101\pi}{5}\right) = \frac{\sqrt{5}+1}{4}$$

إذن

تمرين 24

حل في \mathbb{R} المعادلات التالية :

$$* \cos x = \frac{\sqrt{2}}{2}$$

$$* 2\sin\left(x - \frac{\pi}{3}\right) = 1$$

$$* \sqrt{3} - 2\cos 2x = 0$$

$$* \tan\left(2x - \frac{\pi}{6}\right) = 1$$

$$= \cos\left(\frac{\pi}{2} + \frac{\pi}{5}\right)$$

$$= -\sin\frac{\pi}{5}$$

$$\boxed{\cos\left(\frac{7\pi}{10}\right) = -\frac{\sqrt{5}+1}{4}}$$

$$\sin\left(\frac{-3\pi}{10}\right) = -\sin\left(\frac{3\pi}{10}\right)$$

$$= -\sin\left(\frac{5\pi - 2\pi}{10}\right)$$

$$= -\sin\left(\frac{\pi}{2} - \frac{\pi}{5}\right)$$

$$= -\cos\frac{\pi}{5}$$

$$\boxed{\sin\left(\frac{-3\pi}{10}\right) = -\frac{\sqrt{5}+1}{4}}$$

$$\tan\left(\frac{3\pi}{10}\right) = \frac{\sin\frac{3\pi}{10}}{\cos\frac{3\pi}{10}} = \frac{\cos\frac{\pi}{5}}{\sin\frac{\pi}{5}}$$

$$= \frac{\frac{\sqrt{5}+1}{4}}{\frac{\sqrt{10-2\sqrt{5}}}{4}}$$

$$= \frac{\sqrt{5}+1}{\sqrt{10-2\sqrt{5}}} = \frac{(\sqrt{5}+1) \times \sqrt{10+2\sqrt{5}}}{\sqrt{10-2\sqrt{5}} \times \sqrt{10+2\sqrt{5}}}$$

$$= \frac{(\sqrt{5}+1) \times \sqrt{2} \times \sqrt{5+\sqrt{5}}}{\sqrt{80}}$$

$$= \frac{(\sqrt{5}+1) \sqrt{2} \times \sqrt{5+\sqrt{5}}}{4\sqrt{5}}$$

$$= \frac{(\sqrt{5}+1) \sqrt{2} \times \sqrt{5} \times \sqrt{5+\sqrt{5}}}{20}$$

$$= \frac{(\sqrt{5}+1) \cdot \sqrt{2} \times \sqrt{5} \times \sqrt{5+\sqrt{5}}}{20}$$

$$\cos 2x = \cos \frac{\pi}{6} \quad \text{إذن}$$

$$\begin{cases} 2x = \frac{\pi}{6} + 2k\pi \\ 2x = -\frac{\pi}{6} + 2k\pi \end{cases} \quad \text{أي أن } k \in \mathbf{Z}$$

$$\begin{cases} x = \frac{\pi}{12} + k\pi \\ x = -\frac{\pi}{12} + k\pi \end{cases} \quad \text{أي أن } k \in \mathbf{Z}$$

$$S = \left\{ \frac{\pi}{12} + k\pi / k \in \mathbf{Z} \right\} \cup \left\{ -\frac{\pi}{12} + k\pi / k \in \mathbf{Z} \right\}$$

$$\tan\left(2x - \frac{\pi}{6}\right) = 1 \quad \text{لدينا}$$

$$\tan\left(2x - \frac{\pi}{6}\right) = \tan \frac{\pi}{4} \quad \text{أي أن}$$

$$2x - \frac{\pi}{6} = \frac{\pi}{4} + k\pi \quad \text{إذن}$$

$$k \in \mathbf{Z} \quad \text{مع}$$

$$2x = \frac{\pi}{4} + \frac{\pi}{6} + k\pi \quad \text{إذن}$$

$$k \in \mathbf{Z}, 2x = \frac{5\pi}{12} + k\pi$$

$$k \in \mathbf{Z}, x = \frac{5\pi}{24} + \frac{k\pi}{2} \quad \text{إذن}$$

$$S = \left\{ \frac{5\pi}{24} + \frac{k\pi}{2} / k \in \mathbf{Z} \right\}$$

تمرين 25

حل في \mathbb{R} المعادلات التالية :

$$* 2\cos 3x = -\sqrt{3}$$

$$* \sqrt{2} + 2\sin\left(x - \frac{\pi}{4}\right) = 0$$

$$* \sqrt{3} + \tan 2x = 0$$

الجواب :

$$\cos x = \frac{\sqrt{2}}{2} \quad \text{لدينا}$$

$$\cos x = \cos \frac{\pi}{4} \quad \text{تكافى}$$

$$\begin{cases} x = \frac{\pi}{4} + 2k\pi \\ x = -\frac{\pi}{4} + 2k\pi \end{cases} \quad k \in \mathbf{Z} \quad \text{أي أن}$$

إذن

$$S = \left\{ \frac{\pi}{4} + 2k\pi / k \in \mathbf{Z} \right\} \cup \left\{ -\frac{\pi}{4} + 2k\pi / k \in \mathbf{Z} \right\}$$

$$2\sin\left(x - \frac{\pi}{3}\right) = 1 \quad \text{لدينا}$$

$$\sin\left(x - \frac{\pi}{3}\right) = \frac{1}{2} \quad \text{تكافى}$$

$$\sin\left(x - \frac{\pi}{3}\right) = \sin \frac{\pi}{6} \quad \text{أي}$$

$$\begin{cases} x - \frac{\pi}{3} = \frac{\pi}{6} + 2k\pi \\ x - \frac{\pi}{3} = \pi - \frac{\pi}{6} + 2k\pi \end{cases} \quad k \in \mathbf{Z} \quad \text{إذن}$$

$$\begin{cases} x = \frac{\pi}{3} + \frac{\pi}{6} + 2k\pi \\ x = \frac{5\pi}{6} + \frac{\pi}{3} + 2k\pi \end{cases} \quad k \in \mathbf{Z} \quad \text{إذن}$$

$$\begin{cases} x = \frac{\pi}{2} + 2k\pi \\ x = \frac{7\pi}{6} + 2k\pi \end{cases} \quad k \in \mathbf{Z} \quad \text{إذن}$$

إذن

$$S = \left\{ \frac{\pi}{2} + 2k\pi / k \in \mathbf{Z} \right\} \cup \left\{ \frac{7\pi}{6} + 2k\pi / k \in \mathbf{Z} \right\}$$

$$\sqrt{3} - 2\cos 2x = 0 \quad \text{لدينا}$$

$$-2\cos 2x = -\sqrt{3} \quad \text{أي أن}$$

$$\cos 2x = \frac{\sqrt{3}}{2}$$

$$x = 2k\pi \text{ أو } x = \frac{3\pi}{2} + 2k\pi \quad \text{إذن}$$

إذن

$$S = \left\{ 2k\pi / k \in \mathbb{Z} \right\} \cup \left\{ \frac{3\pi}{2} + 2k\pi / k \in \mathbb{Z} \right\}$$

$$\sqrt{3} + \tan 2x = 0 \quad \text{لدينا}$$

$$\tan 2x = -\sqrt{3} \quad \text{تكافى}$$

$$\tan 2x = \tan\left(-\frac{\pi}{3}\right) \quad \text{أي أن}$$

$$k \in \mathbb{Z}, 2x = -\frac{\pi}{3} + k\pi \quad \text{أي أن}$$

$$x = -\frac{\pi}{6} + \frac{k\pi}{2} \quad \text{إذن}$$

$$S = \left\{ -\frac{\pi}{6} + \frac{k\pi}{2} / k \in \mathbb{Z} \right\} \quad \text{إذن}$$

تمرين 26:

1- حل في المجال $[0 ; 2\pi]$ المعادلة :

$$2\cos(x + \frac{\pi}{3}) = 1$$

2- حل في المجال $[-\pi ; 2\pi]$ المعادلة :

$$2\sin\frac{x}{2} = \sqrt{2}$$

3- حل في المجال $[-\pi ; \pi]$ المعادلة :

$$\cos^2 x + \cos x = 0$$

4- حل في المجال $[0 ; 3\pi]$ المعادلة :

$$\sin^2 x - 2\sin x = 0$$

الجواب :

$$2\cos(x + \frac{\pi}{3}) = 1 \quad \text{لدينا}$$

$$\cos(x + \frac{\pi}{3}) = \frac{1}{2} \quad \text{أي أن}$$

$$\cos(x + \frac{\pi}{3}) = \cos \frac{\pi}{3} \quad \text{لدينا}$$

الجواب :

$$2\cos 3x = -\sqrt{3} \quad \text{لدينا}$$

$$\cos 3x = -\frac{\sqrt{3}}{2} \quad \text{أي أن}$$

$$\cos 3x = -\cos \frac{\pi}{6} \quad \text{إذن}$$

$$\cos 3x = \cos\left(\pi - \frac{\pi}{6}\right) \quad \text{أي أن}$$

$$\cos 3x = \cos \frac{5\pi}{6} \quad \text{إذن}$$

$$\begin{cases} 3x = \frac{5\pi}{6} + 2k\pi \\ 3x = -\frac{5\pi}{6} + 2k\pi \end{cases} \quad \text{أي أن} \quad k \in \mathbb{Z} \quad \text{أو}$$

$$\begin{cases} x = \frac{5\pi}{18} + \frac{2k\pi}{3} \\ x = -\frac{5\pi}{18} + \frac{2k\pi}{3} \end{cases} \quad \text{أي أن} \quad k \in \mathbb{Z} \quad \text{مع}$$

$$S = \left\{ \frac{5\pi}{18} + \frac{2k\pi}{3} / k \in \mathbb{Z} \right\} \cup \left\{ -\frac{5\pi}{18} + \frac{2k\pi}{3} / k \in \mathbb{Z} \right\}$$

$$\sqrt{2} + 2\sin\left(x - \frac{\pi}{4}\right) = 0 \quad \text{لدينا}$$

$$2\sin\left(x - \frac{\pi}{4}\right) = -\sqrt{2} \quad \text{تكافى}$$

$$\sin\left(x - \frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2} \quad \text{أي أن}$$

$$\sin\left(x - \frac{\pi}{4}\right) = -\sin\frac{\pi}{4} \quad \text{أي أن}$$

$$\sin\left(x - \frac{\pi}{4}\right) = \sin\left(-\frac{\pi}{4}\right) \quad \text{إذن}$$

$$x - \frac{\pi}{4} = \pi + \frac{\pi}{4} + 2k\pi \quad \text{إذن}$$

$$k \in \mathbb{Z}, x - \frac{\pi}{4} = -\frac{\pi}{4} + 2k\pi \quad \text{أو}$$

$$\left\{ \begin{array}{l} \frac{x}{2} = \frac{\pi}{4} + 2k\pi \\ \frac{x}{2} = \pi - \frac{\pi}{4} + 2k\pi \end{array} \right. \quad \text{أي أن } k \in \mathbb{Z}$$

$$\left\{ \begin{array}{l} x = \frac{\pi}{2} + 4k\pi \\ x = \frac{3\pi}{2} + 4k\pi \end{array} \right. \quad \text{أي أن } k \in \mathbb{Z}$$

$x \in [-\pi; 2\pi]$ و $x = \frac{\pi}{2} + 4k\pi$ لدينا *

$$-\pi \leq \frac{\pi}{2} + 4k\pi \leq 2\pi \quad \text{إذن}$$

$$-1 \leq \frac{1}{2} + 4k \leq 2 \quad \text{أي أن}$$

$$-\frac{3}{2} \leq 4k \leq \frac{3}{2}$$

$$-\frac{3}{8} \leq k \leq \frac{3}{8} \quad \text{إذن}$$

$$k = 0 \quad \text{و بما أن } k \in \mathbb{Z}$$

$$x = \frac{\pi}{2} \quad \text{و منه}$$

$$x \in [-\pi; 2\pi] \quad \text{و } x = \frac{3\pi}{2} + 4k\pi \quad \text{لدينا} *$$

$$-\pi \leq \frac{3\pi}{2} + 4k\pi \leq 2\pi \quad \text{إذن}$$

$$-1 \leq \frac{3}{2} + 4k \leq 2 \quad \text{أي أن}$$

$$-\frac{5}{2} \leq 4k \leq \frac{1}{2} \quad \text{و منه}$$

$$-\frac{5}{8} \leq k \leq \frac{1}{8}$$

$$k = 0 \quad \text{و بما أن } k \in \mathbb{Z}$$

$$x = \frac{3\pi}{2} \quad \text{و منه}$$

$$S = \left\{ \frac{3\pi}{2}; \frac{\pi}{2} \right\} \quad \text{إذن}$$

$$\cos^2 x + \cos x = 0 \quad \text{لدينا - 3}$$

$$\cos x(\cos x + 1) = 0 \quad \text{تكافي}$$

$$\cos x = 0 \quad \text{أي أن} \quad \cos x + 1 = 0$$

$$\cos x = 0 \quad \text{أي أن} \quad \cos x = -1$$

$$x + \frac{\pi}{3} = \frac{\pi}{3} + 2\pi k \quad \text{أي أن}$$

$$x + \frac{\pi}{3} = -\frac{\pi}{3} + 2\pi k \quad \text{أو}$$

حيث

$$\left\{ \begin{array}{l} x = 2k\pi \\ x = -\frac{2\pi}{3} + 2k\pi \end{array} \right. \quad \text{أي أن } k \in \mathbb{Z}$$

$$x \in [0; 2\pi] \quad \text{و } x = 2\pi k \quad \text{لدينا *}$$

$$0 \leq 2\pi k \leq 2\pi \quad \text{إذن}$$

$$0 \leq 2k \leq 2 \quad \text{أي أن}$$

$$0 \leq k \leq 1 \quad \text{و منه}$$

$$k = 0 \quad \text{أو} \quad k = 1 \quad \text{فإن } k \in \mathbb{Z}$$

$$x = 0 \quad \text{إذا كان} \quad k = 0 \quad \text{فإن}$$

$$x = 2\pi \quad \text{إذا كان} \quad k = 1 \quad \text{فإن}$$

$$x = -\frac{2\pi}{3} + 2k\pi \quad \text{لدينا كذلك *}$$

$$x \in [0; 2\pi] \quad \text{و}$$

$$0 \leq -\frac{2\pi}{3} + 2k\pi \leq 2\pi \quad \text{إذن}$$

$$0 \leq -\frac{2}{3} + 2k \leq 2 \quad \text{إذن}$$

$$\frac{2}{3} \leq 2k \leq \frac{8}{3} \quad \text{و منه}$$

$$\frac{1}{3} \leq k \leq \frac{4}{3} \quad \text{إذن}$$

$$k = 1 \quad \text{و بما أن } k \in \mathbb{Z} \quad \text{إذن}$$

$$x = \frac{4\pi}{3} \quad \text{إذا كان} \quad k = 1 \quad \text{فإن}$$

$$S = \left\{ 0; 2\pi, \frac{4\pi}{3} \right\} \quad \text{إذن}$$

$$2\sin \frac{x}{2} = \sqrt{2} \quad \text{لدينا - 2}$$

$$\sin \frac{x}{2} = \frac{\sqrt{2}}{2}$$

$$\sin \frac{x}{2} = \sin \frac{\pi}{4} \quad \text{أي أن}$$

$$0 \leq k \leq 3$$

إذن $k = 3$ أو $k = 2$ أو $k = 1$ أو $k = 0$

إذا كان $k = 0$ فإن $x = 0$

إذا كان $k = 1$ فإن $x = \pi$

إذا كان $k = 2$ فإن $x = 2\pi$

إذا كان $k = 3$ فإن $x = 3\pi$

$$S = \{0; \pi; 2\pi; 3\pi\}$$

تمرين 27

(1) - حل في المجال $[0, 2\pi]$

$$\cos(x) = \sin(x)$$

(2) - حل في المجال $[-\pi, 0]$

$$\cos\left(x - \frac{\pi}{3}\right) = -\sin(x)$$

(3) - حل في المجال $[-\frac{\pi}{2}, \frac{\pi}{2}]$

$$\tan(x) = \sin(x)$$

الجواب :

$$\cos(x) = \sin(x) \quad \text{لدينا} \quad (1)$$

$$\cos(x) = \cos\left(\frac{\pi}{2} - x\right) \quad \text{تكافي}$$

$$x = \frac{\pi}{2} - x + 2k\pi \quad \text{أي أن}$$

$$k \in \mathbf{Z}, \quad x = -\frac{\pi}{2} + x + 2k\pi \quad \text{أو}$$

$$x + x = \frac{\pi}{2} + 2k\pi \quad \text{أي أن}$$

$$k \in \mathbf{Z}, \quad x - x = -\frac{\pi}{2} + 2k\pi \quad \text{أو}$$

$$2x = \frac{\pi}{2} + 2k\pi \quad \text{أي أن}$$

$$x = \frac{\pi}{2} + k\pi \quad \text{أو} \quad x = \pi + 2k\pi$$

إذن $k \in \mathbf{Z}$ مع

$$x \in [-\pi; \pi] \quad \text{لدينا}^*$$

$$-\pi \leq \frac{\pi}{2} + k\pi \leq \pi \quad \text{إذن}$$

$$-1 \leq \frac{1}{2} + k \leq 1$$

$$-\frac{3}{2} \leq k \leq \frac{1}{2}$$

$$k = 0 \quad \text{أو} \quad k = -1 \quad \text{إذن} \quad k \in \mathbf{Z}$$

$$x = \frac{-\pi}{2} \quad \text{إذن} \quad k = -1$$

$$x = \frac{\pi}{2} \quad \text{إذن} \quad k = 0$$

$$x \in [-\pi; \pi] \quad \text{لدينا}^* \quad x = \pi + 2k\pi$$

$$-\pi \leq \pi + 2k\pi \leq \pi \quad \text{إذن}$$

$$-2 \leq 2k \leq 0 \quad \text{أي}$$

$$-1 \leq k \leq 0$$

$$k = -1 \quad \text{أو} \quad k = 0 \quad \text{إذن} \quad k \in \mathbf{Z}$$

$$x = \pi \quad \text{إذن} \quad k = 0 \quad \text{فإن}$$

$$x = -\pi \quad \text{فإن} \quad k = -1 \quad \text{إذن}$$

$$S = \left\{ \frac{-\pi}{2}; \frac{\pi}{2}; \pi; -\pi \right\} \quad \text{إذن}$$

$$\sin^2 x - 2\sin x = 0 \quad \text{لدينا}^* \quad 4$$

$$\sin x (\sin x - 2) = 0 \quad \text{تكافي}$$

$$\sin x = 0 \quad \text{أي أن} \quad \sin x = 2 \quad \text{أي أن}$$

$$\text{لا يمكن لأن } -1 \leq \sin x \leq 1 \quad \text{أو مع} \quad x = k\pi$$

$$k \in \mathbf{Z}$$

$$x \in [0; 3\pi] \quad \text{لدينا}^* \quad x = k\pi$$

$$0 \leq k\pi \leq 3\pi \quad \text{إذن}$$

$-\frac{11}{12} \leq k \leq \frac{1}{12}$	إذن
$k = 0 \quad k \in \mathbf{Z}$	بما أن 0 فإن
$S = \left\{-\frac{11}{12}\right\}$	إذن
(E) : $\tan(x) = \sin(x)$	- لدينا (1)
$x \neq \frac{\pi}{2} + k\pi$	تكافى $x \in D_E$
$\frac{\sin x}{\cos x} = \sin(x)$	لدينا (E) تكافى
$\sin x = \cos x \cdot \sin x$	أي أن
$\sin x - \cos x \cdot \sin x = 0$	أي أن
$\sin x(1 - \cos x) = 0$	أي أن
$\sin x = 0 \quad 1 - \cos x = 0$	تكافى
$\sin x = 0 \quad \text{أو} \quad \cos x = 1$	و منه
$x = k\pi \quad \text{أو} \quad x = 2k\pi$	أي أن
$k \in \mathbf{Z}, \quad x = k\pi$	إذن
$x = k\pi \quad \text{و} \quad x \in \left]-\frac{\pi}{2}, \frac{\pi}{2}\right[$	بما أن
$-\frac{\pi}{2} \leq k\pi \leq \frac{\pi}{2}$	فإن
$-\frac{1}{2} \leq k \leq \frac{1}{2}$	
$k = 0 \quad \text{فإن} \quad k \in \mathbf{Z}$	بما أن
$x = 0$	إذن
$S = \{0\}$	إذن

$0 = -\frac{\pi}{2} + 2k\pi$	لما $0 = -\frac{\pi}{2} + 2k\pi$
$x = \frac{\pi}{4} + k\pi$	إذن
$x \in [0, 2\pi] \quad \text{و} \quad x = \frac{\pi}{4} + k\pi$	لدينا
$0 \leq \frac{\pi}{4} + k\pi \leq 2\pi$	إذن
$0 \leq \frac{1}{4} + k \leq 2$	
$-\frac{1}{4} \leq k \leq \frac{7}{4}$	أي أن
$k = 1 \quad \text{أو} \quad k = 0 \quad k \in \mathbf{Z}$	بما أن $0 \quad \text{فإن} \quad k \in \mathbf{Z}$
$x = \frac{\pi}{4}$	إذا كان 0 $\quad \text{فإن} \quad k = 0$
$x = \frac{5\pi}{4}$	إذا كان 1 $\quad \text{فإن} \quad k = 1$
$S = \left\{\frac{\pi}{4}, \frac{5\pi}{4}\right\}$	إذن
$\cos\left(x - \frac{\pi}{3}\right) = -\sin(x)$	- لدينا (2)
$\cos\left(x - \frac{\pi}{3}\right) = \sin(-x)$	تكافى
$\cos\left(x - \frac{\pi}{3}\right) = \cos\left(\frac{\pi}{2} + x\right)$	أي أن
$\begin{cases} x - \frac{\pi}{3} = \frac{\pi}{2} + x + 2k\pi \\ x - \frac{\pi}{3} = -\frac{\pi}{2} - x + 2k\pi \end{cases}, \quad k \in \mathbf{Z}$	إذن
$x - x = \frac{\pi}{2} + \frac{\pi}{3} + 2k\pi$	تكافى
$x + x = -\frac{\pi}{2} + \frac{\pi}{3} + 2k\pi$	
$0 = \frac{5\pi}{6} + 2k\pi, \quad k \in \mathbf{Z}$	
$2x = -\frac{\pi}{6} + 2k\pi$	إذن
$k \in \mathbf{Z}, \quad x = -\frac{\pi}{12} + k\pi$	إذن
$x = -\frac{\pi}{12} + k\pi \quad \text{و} \quad x \in [-\pi, 0]$	بما أن
$-\pi \leq -\frac{\pi}{12} + k\pi \leq 0$	فإن
$-1 \leq -\frac{1}{12} + k \leq 0$	

تمرين 28:

$$(E') 2\sin^2x + (2 - \sqrt{2})\sin x - \sqrt{2} = 0$$

$$X = \sin x \quad \text{لنسع}$$

$$\text{إذن } (E) \text{ تكافى} \quad 2X^2 + (2 - \sqrt{2})X - \sqrt{2} = 0$$

$$\Delta = (2 - \sqrt{2})^2 - 4 \times 2(-\sqrt{2})$$

$$= 2^2 - 4\sqrt{2} + \sqrt{2}^2 + 8\sqrt{2}$$

$$= 2^2 + 4\sqrt{2} + \sqrt{2}^2$$

$$= (2 + \sqrt{2})^2$$

$$\sqrt{\Delta} = 2 + \sqrt{2}$$

إذن

$$X = \frac{-2 + \sqrt{2} + 2 + \sqrt{2}}{4} \quad \text{أو} \quad X = \frac{-2 + \sqrt{2} - 2 - \sqrt{2}}{4}$$

$$X = \frac{\sqrt{2}}{2} \quad \text{أي أن} \quad X = -1$$

$$\sin x = \frac{\sqrt{2}}{2} \quad \text{أو} \quad \sin x = -1$$

$$\sin x = \sin \frac{\pi}{4} \quad \text{أي أن} \quad \sin x = -1$$

$$x = \frac{\pi}{4} + 2k\pi \quad \text{أو} \quad x = -\frac{\pi}{2} + 2k\pi$$

$$x = \frac{3\pi}{4} + 2k\pi \quad \text{أي أن}$$

إذن

$$S = \left\{ \frac{\pi}{4} + 2k\pi / k \in \mathbb{Z} \right\} \cup \left\{ \frac{3\pi}{4} + 2k\pi / k \in \mathbb{Z} \right\} \\ \cup \left\{ \frac{-\pi}{2} + 2k\pi / k \in \mathbb{Z} \right\}$$

- لدينا

$$(E'') \tan^2 x + (\sqrt{3} - 1)\tan x - \sqrt{3} = 0$$

$$k \in \mathbb{Z} \quad x \neq \frac{\pi}{2} + k\pi \quad \text{تكافى} \quad x \in D(E'')$$

$$X = \tan x \quad \text{لنسع}$$

$$X^2 + (\sqrt{3} - 1)X - \sqrt{3} = 0 \quad \text{إذن } (E'') \text{ تكافى}$$

(1) - حل في \mathbb{R} المعادلة :

$$(E) 2\cos^2 x - 5\cos x - 3 = 0$$

(2) - حل في \mathbb{R} المعادلة :

$$(E') 2\sin^2 x + (2 - \sqrt{2})\sin x - \sqrt{2} = 0$$

(3) - حل في \mathbb{R} المعادلة :

$$(E'') \tan^2 x + (\sqrt{3} - 1)\tan x - \sqrt{3} = 0$$

الجواب :

$$2\cos^2 x - 5\cos x - 3 = 0 \quad (1) \quad \text{- لدينا}$$

$$X = \cos x \quad \text{لنسع}$$

$$2X^2 - 5X - 3 = 0 \quad \text{إذن } (E) \text{ تكافى}$$

$$\Delta = 25 + 24 = 49$$

$$\sqrt{\Delta} = 7 \quad \text{لدينا}$$

$$X = \frac{5+7}{4} \quad \text{أي أن} \quad X = \frac{5-7}{4}$$

$$X = 3 \quad \text{أي أن} \quad X = -\frac{1}{2}$$

$$\cos x = 3 \quad \text{أي أن} \quad \cos x = -\frac{1}{2}$$

لا يمكن $\cos x = 3$

$$\cos x = \cos \frac{2\pi}{3} \quad \text{أي أن}$$

$$\begin{cases} x = \frac{2\pi}{3} + 2k\pi \\ \text{أي} \\ x = -\frac{2\pi}{3} + 2k\pi \end{cases} \quad k \in \mathbb{Z}$$

$$S = \left\{ \frac{2\pi}{3} + 2k\pi / k \in \mathbb{Z} \right\} \cup \left\{ -\frac{2\pi}{3} + 2k\pi / k \in \mathbb{Z} \right\}$$

- لدينا



الجواب :

$$(I) \quad \begin{cases} x \in [0, 2\pi] \\ -1 + 2\cos x \geq 0 \end{cases}$$

نعتبر المعادلة : $-1 + 2\cos x = 0$

$$\cos x = \frac{1}{2} \quad \text{أي أن}$$

$$\cos x = \cos \frac{\pi}{3} \quad \text{إذن}$$

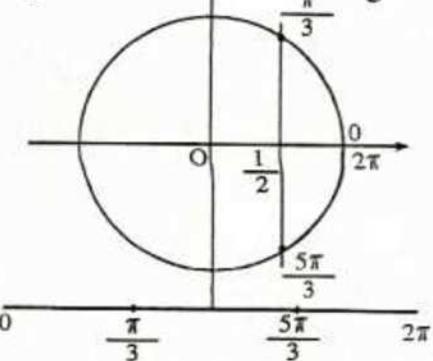
إذن :

$$x = \frac{\pi}{3} + 2k\pi \quad \text{أو} \quad x = -\frac{\pi}{3} + 2k\pi$$

حيث $k \in \mathbb{Z}$

وبما أن $x \in [0, 2\pi]$ فإن :

$$x = \frac{\pi}{3} \quad \text{أو} \quad x = \frac{5\pi}{3}$$



(I) تكافىء أي أن $\cos x \geq \frac{1}{2}$

$$0 \leq x \leq \frac{\pi}{3} \quad \text{أو} \quad \frac{5\pi}{3} \leq x \leq 2\pi$$

$$S = \left[0, \frac{\pi}{3}\right] \cup \left[\frac{5\pi}{3}, 2\pi\right] \quad \text{إذن}$$

2 - لدينا

$$(I) \quad \begin{cases} x \in [-\pi, \pi] \\ -2\sin x + \sqrt{2} < 0 \end{cases}$$

$$(I) \quad \begin{cases} x \in [-\pi, \pi] \\ \sin x > \frac{\sqrt{2}}{2} \end{cases} \quad \text{تكافىء :}$$

$$\Delta = (\sqrt{3} - 1)^2 + 4\sqrt{3}$$

$$= \sqrt{3}^2 - 2\sqrt{3} + 1^2 + 4\sqrt{3}$$

$$= \sqrt{3}^2 + 2\sqrt{3} + 1^2$$

$$= (\sqrt{3} + 1)^2$$

$$\sqrt{\Delta} = \sqrt{3} + 1$$

إذن

$$X = \frac{-\sqrt{3} + 1 + \sqrt{3} + 1}{2} = 1 \quad \text{و منه}$$

$$X = \frac{-\sqrt{3} + 1 - \sqrt{3} - 1}{2} = -\sqrt{3} \quad \text{أو}$$

$$\tan x = 1 \quad \text{أو} \quad \tan x = -\sqrt{3} \quad \text{إذن}$$

$$\tan x = \tan \frac{\pi}{4} \quad \text{أي أن} \quad \tan x = -\tan \frac{\pi}{3}$$

$$\tan x = \tan \frac{\pi}{4} \quad \text{أي أن} \quad \tan x = \tan \left(-\frac{\pi}{3}\right)$$

$$x = \frac{\pi}{4} + k\pi \quad \text{و منه}$$

أو $k \in \mathbb{Z}$

$$x = -\frac{\pi}{3} + k\pi$$

$$S = \left\{ \frac{\pi}{4} + k\pi / k \in \mathbb{Z} \right\} \cup \left\{ -\frac{\pi}{3} + k\pi / k \in \mathbb{Z} \right\}$$

تمرين 29:

1 - حل في المجال : $[0, 2\pi]$

المتراجحة : $-1 + 2\cos x \geq 0$

2 - حل في المجال : $[0, 2\pi]$ المتراجحة :

$-2\sin x + \sqrt{2} < 0$

3 - حل في المجال : $\left]-\frac{\pi}{2}, \frac{\pi}{2}\right[$

المتراجحة : $\sqrt{3} - \tan x \leq 0$

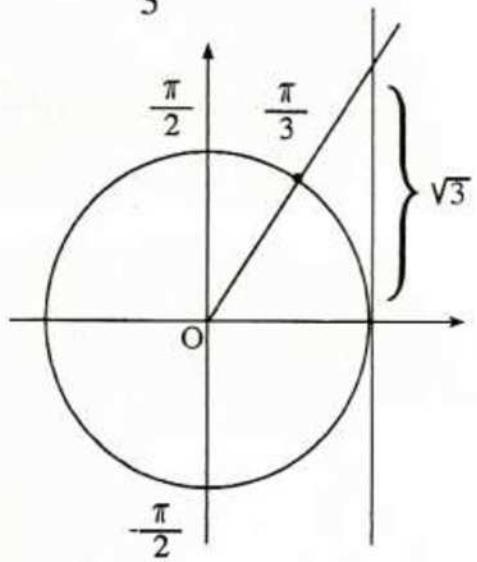
$$\tan x = \sqrt{3} \quad \text{تكافىء :}$$

$$\tan x = \tan \frac{\pi}{3} \quad \text{أى أن}$$

$$x = \frac{\pi}{3} + k\pi, k \in \mathbf{Z}$$

بما أن $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2} \right]$ فإن :

$$x = \frac{\pi}{3}$$



$$\tan x \geq \sqrt{3} \quad (\text{I''}) \quad \text{تكافىء}$$

$$\frac{\pi}{3} \leq x < \frac{\pi}{2} \quad \text{أى أن}$$

$$S = \left[\frac{\pi}{3}, \frac{\pi}{2} \right] \quad \text{إذن}$$

تمرين 30:

1 - حل في المجال $[0, 2\pi]$

$$2\cos x(2x) \geq \sqrt{3} \quad \text{المتراجحة :}$$

2 - حل في المجال $[-\pi, \pi]$

$$\sin(2x + \frac{\pi}{4}) < \frac{\sqrt{2}}{2} \quad \text{المتراجحة :}$$

3 - حل في المجال $[0, \pi]$

$$\tan \frac{x}{2} < 1$$

$$\sin x = \frac{\sqrt{2}}{2} \quad \text{نعتبر المعادلة :}$$

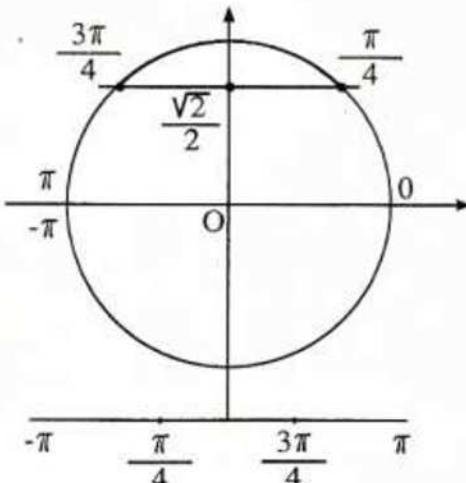
$$\sin x = \frac{\pi}{4} \quad \text{أى أن}$$

إذن :

$$x = \frac{\pi}{4} + 2k\pi \quad \text{أو} \quad x = \frac{3\pi}{4} + 2k\pi \quad k \in \mathbf{Z} \quad \text{حيث}$$

وبما أن $x \in [-\pi, \pi]$ فإن :

$$x = \frac{\pi}{4} \quad \text{أو} \quad x = \frac{3\pi}{4}$$



$$\sin x > \frac{\sqrt{2}}{2} \quad (\text{I'}) \quad \text{تكافىء}$$

$$\frac{\pi}{4} < x < \frac{3\pi}{4} \quad \text{أى أن}$$

$$S = \left[\frac{\pi}{4}, \frac{3\pi}{4} \right] \quad \text{إذن}$$

2 - لدينا

$$(\text{I''}) \quad \begin{cases} x \in \left[-\frac{\pi}{2}, \frac{\pi}{2} \right] \\ \sqrt{3} - \tan x \leq 0 \end{cases}$$

$$\sqrt{3} - \tan x = 0 \quad \text{نعتبر المعادلة :}$$

إذن : $0 < x < \frac{\pi}{12}$ أو $\frac{11\pi}{12} < x < \pi$

إذن : $S = \left[\frac{\pi}{4}, \frac{3\pi}{4}\right] \cup \left[\frac{11\pi}{12}, \pi\right]$

- لدينا 2

$$(I) \begin{cases} x \in [-\pi, \pi] \\ \sin(2x + \frac{\pi}{4}) < \frac{\sqrt{2}}{2} \end{cases}$$

نعتبر المعادلة : $X = 2x + \frac{\pi}{4}$

لدينا : $-\pi \leq x \leq \pi$

أو $-2\pi \leq 2x \leq -2\pi$

$-\frac{7\pi}{4} \leq 2x + \frac{\pi}{4} \leq \frac{9\pi}{4}$: أي أن

$$(I) : \begin{cases} X \in \left[-\frac{7\pi}{4}, \frac{9\pi}{4}\right] \\ \sin X < \frac{\sqrt{2}}{2} \end{cases}$$

نعتبر المعادلة : $\sin X = \frac{\sqrt{2}}{2}$

$\sin X = \sin \frac{\pi}{4}$

$X = \frac{\pi}{4} + 2k\pi$ أو $X = \frac{3\pi}{4} + 2k\pi$

$k \in \mathbb{Z}$ مع

بما أن $X \in \left[-\frac{7\pi}{4}, \frac{9\pi}{4}\right]$ فإن

$X = \frac{\pi}{4}$ أو $X = -\frac{7\pi}{4}$ أو $X = \frac{9\pi}{4}$ أو

الجواب :

$$(I) : \begin{cases} x \in [0, \pi] \\ 2\cos x (2x) \geq \sqrt{3} \end{cases}$$

$X = 2x$ لنضع

$$X \in [0, 2\pi] \quad x \in [0, \pi]$$

$$\begin{cases} X \in [0, 2\pi] \\ 2\cos X \geq \sqrt{3} \end{cases} : (I)$$

نعتبر المعادلة : $2\cos X = \sqrt{3}$

$\cos X = \frac{\sqrt{3}}{2}$ أي أن

$\cos X = \cos \frac{\pi}{6}$ إذن

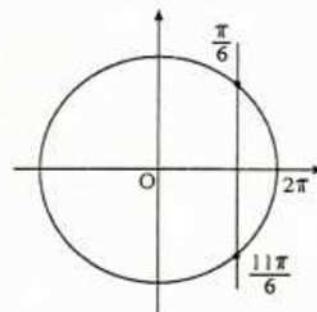
إذن :

$X = \frac{\pi}{6} + 2k\pi$ أو $X = -\frac{\pi}{6} + 2k\pi$

$k \in \mathbb{Z}$ مع

بما أن $X \in [0, 2\pi]$ فإن :

$X = \frac{\pi}{6}$ أو $X = \frac{11\pi}{6}$



$\sin X \geq \frac{\sqrt{3}}{2}$ تكافىء (I')

أي أن $0 < X < \frac{\pi}{6}$ أو $\frac{11\pi}{6} < X < 2\pi$:

أي أن $0 < 2x < \frac{\pi}{6}$ أو $\frac{11\pi}{6} < 2x < 2\pi$:

$$X = \frac{x}{2} \quad \text{لنضع}$$

$$X \in \left[0, \frac{\pi}{2}\right] \quad \text{إذن} \quad x \in [0, \pi]$$

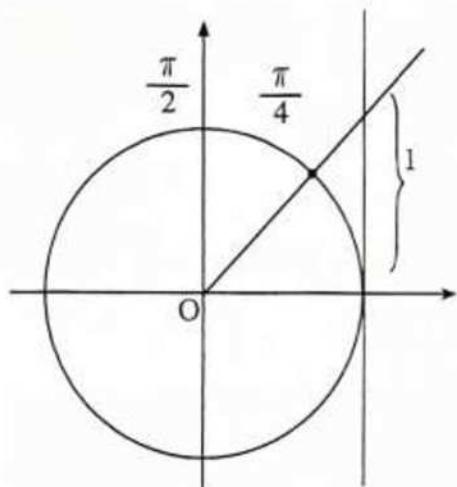
$$\begin{cases} X \in \left[0, \frac{\pi}{2}\right] \\ \tan X < 1 \end{cases} \quad \text{تكافىء (I'')}$$

نعتبر المعادلة : $\tan X = 1$

$$\tan X = \tan \frac{\pi}{4}$$

$$k \in \mathbf{Z}, \quad X = \frac{\pi}{4} + k\pi$$

$$X = \frac{\pi}{4} : \text{فإن } X \in \left[0, \frac{\pi}{2}\right] \quad \text{وبما أن}$$



$\tan X < 1$ تكافىء (I'')

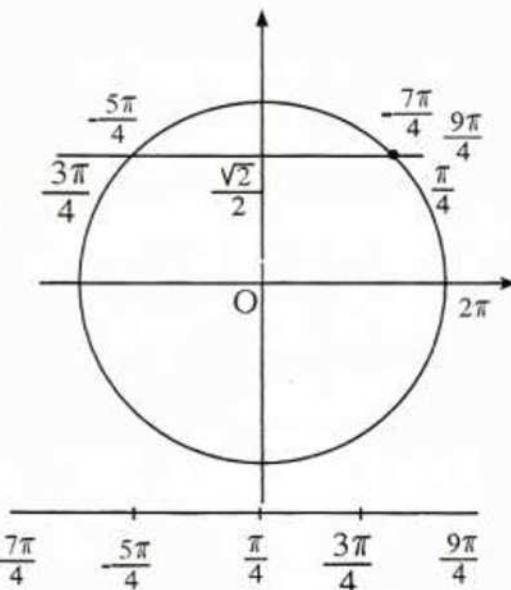
$$0 \leq X < \frac{\pi}{4} \quad \text{أي أن :}$$

$$0 \leq \frac{x}{2} < \frac{\pi}{4} \quad \text{أي أن :}$$

$$0 \leq x < \frac{\pi}{2} \quad \text{أي أن :}$$

$$S = \left[0, \frac{\pi}{2}\right[\quad \text{إذن :}$$

$$X = \frac{3\pi}{4} \quad \text{أو} \quad X = -\frac{5\pi}{4}$$



$\sin X < \frac{\sqrt{2}}{2}$ تكافىء (I')

$$-\frac{5\pi}{4} < X < \frac{\pi}{4} \quad \text{أو} \quad \frac{3\pi}{4} < X < \frac{9\pi}{4}$$

$$\frac{3\pi}{4} < 2x + \frac{\pi}{4} < \frac{9\pi}{4} \quad \text{أي أن :}$$

$$-\frac{5\pi}{4} < 2x + \frac{\pi}{4} < \frac{\pi}{4} \quad \text{أو}$$

$$-\frac{3\pi}{2} < 2x < 0 \quad \text{أو} \quad \frac{\pi}{2} < 2x < 2\pi \quad \text{أي أن :}$$

$$-\frac{3\pi}{2} < x < 0 \quad \text{أو} \quad \frac{\pi}{4} < x < \pi \quad \text{أي أن :}$$

$$S = \left[-\frac{3\pi}{2}, 0\right] \cup \left[\frac{\pi}{4}, \pi\right] \quad \text{إذن :}$$

$$(I'') : \quad \begin{cases} x \in [0, \pi] \\ \sin x \left(\frac{x}{2}\right) < 1 \end{cases} \quad \text{لدينا - 3}$$

حيث $k \in \mathbf{Z}$

$$S = \left\{ \frac{\pi}{6} + 2k\pi / k \in \mathbf{Z} \right\} \cup \left\{ \frac{5\pi}{6} + 2k\pi / k \in \mathbf{Z} \right\} \\ \cup \left\{ \frac{\pi}{2} + 2k\pi / k \in \mathbf{Z} \right\}$$

2 - حلول المعادلة على $Q(x) = 0$

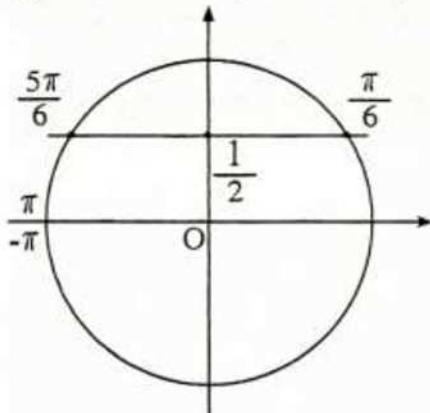
المجال هي $[-\pi, \pi]$

$$x = \frac{\pi}{6} \quad \text{أو} \quad x = \frac{5\pi}{6} \quad \text{أو} \quad x = \frac{\pi}{2}$$

x	$-\pi$	$\frac{\pi}{6}$	$\frac{\pi}{2}$	$\frac{5\pi}{6}$	π
$1 - \sin x$	+	+	○	+	+
$2\sin x - 1$	-	○	+	+	○ -
$Q(x)$	-	○	+	○	-

ملاحظة: $1 \leq \sin x \leq 1$

إذن $0 \leq 1 - \sin x$ أي أن $\sin x \leq 1$



$$\begin{cases} x \in [-\pi, \pi] \\ Q(x) > 1 \end{cases} \quad \text{بـ}$$

تكافىء: $\frac{\pi}{6} < x < \frac{\pi}{2}$ أو $\frac{\pi}{2} < x < \frac{5\pi}{6}$

$$S = \left[\frac{\pi}{6}, \frac{\pi}{2} \right] \cup \left[\frac{\pi}{2}, \frac{5\pi}{6} \right] \quad \text{إذن}$$

تمرين 31:

ليكن لكل x من \mathbb{R}

$$Q(x) = -2\sin^2 x + 3\sin x - 1$$

1 - أ - بين أن لكل x من \mathbb{R} .

$$Q(x) = (2\sin x - 1)(1 - \sin x)$$

ب - حل في \mathbb{R} المعادلة

2 - أ - ادرس إشارة $Q(x)$ على $[-\pi, \pi]$.

المجال.

ب - استنتج حلول المتراجحة

$$Q(x) > 0 \quad \text{على المجال } [-\pi, \pi]$$

الجواب:

$$(2\sin x - 1)(1 - 2\sin x) \quad \text{أ - لدينا}$$

$$= 2\sin x - 2\sin^2 x - 1 + \sin x$$

$$= -2\sin^2 x + 3\sin x - 1$$

$$= Q(x)$$

$$Q(x) = -2\sin^2 x + 3\sin x - 1 \quad \text{إذن:}$$

ب - تكافىء $Q(x) = 0$

$$(2\sin x - 1)(1 - \sin x) = 0$$

أي أن $2\sin x - 1 = 0$ أو $1 - \sin x = 0$

$$\sin x = \frac{1}{2} \quad \text{أي أن} \quad \sin x = 1$$

$$\sin x = \sin \frac{\pi}{6} \quad \text{أي أن} \quad \sin x = 1$$

$$x = \frac{\pi}{6} + 2k\pi \quad \text{أو} \quad x = \frac{5\pi}{6} + 2k\pi$$

$$x = \frac{\pi}{2} + 2k\pi$$

$$\in \mathbf{Z} \left\} \cup \left\{ -\frac{2\pi}{3} + 2k\pi / k \in \mathbf{Z} \right\}$$

نعتبر لـ x من \mathbb{R} تكافيء $P(x) = 0$ - أ - 2

$$x = \frac{\pi}{2} + k\pi \quad \text{أو} \quad x = \frac{2\pi}{3} + 2k\pi$$

$$x = -\frac{2\pi}{3} + 2k\pi \quad \text{أو}$$

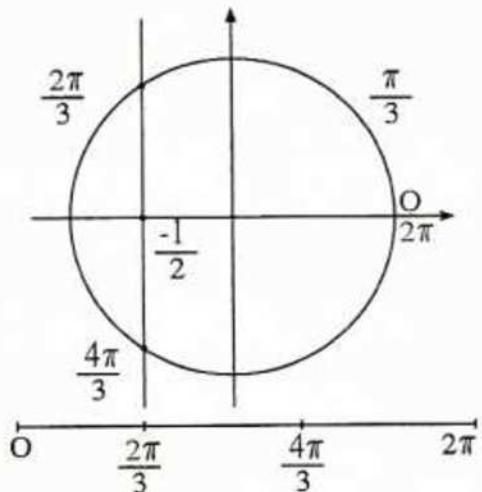
حيث $k \in \mathbf{Z}$

حلول المعادلة $P(x) = 0$ على المجال :

: هي $[0, 2\pi]$

$$x = \frac{\pi}{2} \quad \text{أو} \quad x = \frac{3\pi}{2} \quad \text{أو} \quad x = \frac{2\pi}{3} \quad \text{أو} \quad x = \frac{4\pi}{3}$$

x	0	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	2π
$\cos x$	+	0	-	-	-	0
$2\cos x + 1$	+	+	0	-	0	+
$P(x)$	+	0	-	0	+	0



نعتبر لـ x من \mathbb{R} تكافيء $P(x) \leq 0$

$$\frac{\pi}{2} < x < \frac{2\pi}{3} \quad \text{أو} \quad \frac{4\pi}{3} < x < \frac{3\pi}{2}$$

$$S = \left[\frac{\pi}{2}, \frac{2\pi}{3} \right] \cup \left[\frac{4\pi}{3}, \frac{3\pi}{2} \right] \quad \text{إذن}$$

تمرين 32:

نعتبر لـ x من \mathbb{R}

$$P(x) = 2\cos^2 x + \cos x$$

1 - حل في \mathbb{R} المعادلة : $P(x) = 0$

2 - أدرس إشارة $P(x)$ لـ x من المجال $[0, 2\pi]$

ب - استنتج حلول المتراجحة $P(x) \leq 0$ لـ x من $[0, 2\pi]$

الجواب :

نعتبر لـ x من \mathbb{R} تكافيء $P(x) = 0$ - 1

$$2\cos^2 x + \cos x = 0$$

$$\cos x(2\cos x + 1) = 0$$

$$\cos x = 0 \quad \text{أو} \quad 2\cos x + 1 = 0$$

$$\cos x = 0 \quad \text{أو} \quad \cos x = -\frac{1}{2}$$

$$\cos x = 0 \quad \text{أو} \quad \cos x = -\cos \frac{\pi}{3}$$

$$\cos x = 0 \quad \text{أو} \quad \cos x = \cos \frac{2\pi}{3}$$

$$x = \frac{\pi}{2} + k\pi \quad \text{أو} \quad x = \frac{2\pi}{3} + 2k\pi$$

$$x = -\frac{2\pi}{3} + 2k\pi \quad \text{أو}$$

حيث $k \in \mathbf{Z}$

$$S = \left\{ \frac{\pi}{2} + k\pi / k \in \mathbf{Z} \right\} \cup \left\{ \frac{2\pi}{3} + 2k\pi / k \in \mathbf{Z} \right\}$$

من اقتراح أذ : لخربسي سمير

1

المعادلات التالية : IR

$2 \sin\left(3x - \frac{2\pi}{3}\right) = \sqrt{3}$	$\cos\left(2x - \frac{\pi}{3}\right) = \frac{\sqrt{2}}{2}$	$2 \cos x + 1 = 0$	$2 \sin x + \sqrt{2} = 0$
$2 \sin x + \tan x = 0$	$\cos(3x) \cdot (\sin x - 1) = 0$	$\sin x = \cos x$	$\sin\left(5x - \frac{\pi}{3}\right) = \sin\left(x + \frac{\pi}{3}\right)$
$4 \cos^2 x - 1 = 0$	$3 \tan^2 x - 1 = 0$	$\sin^2 x - 4 = 0$	$\sin\left(x + \frac{\pi}{6}\right) = \cos\left(2x + \frac{\pi}{3}\right)$

2

المتراجحات التالية : $[-\pi, \pi]$ $[0, 2\pi]$

$2 \cos x \leq -\sqrt{3}$	$2 \cos x \leq \sqrt{2}$	$2 \cos x + \sqrt{3} \geq 0$	$2 \cos x \geq 1$
$\cos x \leq -5$	$\cos x \leq 4$	$\cos x \geq 2$	$-\sqrt{2} \leq 2 \cos x \leq 1$
$\tan x \geq 1$	$-1 \leq 2 \sin x \leq \sqrt{2}$	$2 \sin x + \sqrt{2} \geq 0$	$2 \sin x \leq -\sqrt{3}$
$\cos x \geq \cos\left(\frac{5\pi}{8}\right)$	$\sin x \geq \sin\left(-\frac{\pi}{5}\right)$	$\sqrt{3} \tan x \leq -1$	$\tan x < \sqrt{3}$

3

$$I = [0, 3\pi] \quad 2 \sin x - 1 = 0 :$$

$$2 \sin x - 1 \geq 0 : \quad I$$

4

$$J = [0, 5\pi] \quad 2 \cos x + 1 = 0 :$$

$$2 \cos x + 1 > 0 : \quad J$$

5

$$K = [0, 4\pi] \quad \tan x = \sqrt{3} :$$

$$\tan x = \sqrt{3} : \quad K$$

$$\sin x = \sin\left(\frac{-\pi}{4}\right)$$

$$\sin x = \frac{-\sqrt{2}}{2} :$$

$$2 \sin x + \sqrt{2} = 0$$

$$x = \pi - \left(\frac{-\pi}{4}\right) + 2k\pi / k \in Z \quad x = \frac{-\pi}{4} + 2k\pi / k \in Z :$$

$$S = \left\{ \frac{-\pi}{4} + 2k\pi / k \in Z \right\} \cup \left\{ \frac{5\pi}{4} + 2k\pi / k \in Z \right\} :$$

$$\cos x = \cos\left(\frac{2\pi}{3}\right) :$$

$$\cos x = \frac{-1}{2} :$$

$$2 \cos x + 1 = 0$$

$$x = \frac{-2\pi}{3} + 2k\pi / k \in Z \quad x = \frac{2\pi}{3} + 2k\pi / k \in Z :$$

$$S = \left\{ \frac{2\pi}{3} + 2k\pi / k \in Z \right\} \cup \left\{ \frac{-2\pi}{3} + 2k\pi / k \in Z \right\} :$$

$$\cos\left(2x - \frac{\pi}{3}\right) = \cos\left(\frac{\pi}{4}\right) :$$

$$\cos\left(2x - \frac{\pi}{3}\right) = \frac{\sqrt{2}}{2}$$

$$2x - \frac{\pi}{3} = \frac{-\pi}{4} + 2k\pi / k \in Z \quad 2x - \frac{\pi}{3} = \frac{\pi}{4} + 2k\pi / k \in Z :$$

$$2x = \frac{-\pi}{4} + \frac{\pi}{3} + 2k\pi / k \in Z \quad 2x = \frac{\pi}{4} + \frac{\pi}{3} + 2k\pi / k \in Z :$$

$$2x = \frac{\pi}{12} + 2k\pi / k \in Z \quad 2x = \frac{7\pi}{12} + 2k\pi / k \in Z :$$

$$x = \frac{\pi}{24} + k\pi / k \in Z \quad x = \frac{7\pi}{24} + k\pi / k \in Z :$$

$$S = \left\{ \frac{\pi}{24} + k\pi / k \in Z \right\} \cup \left\{ \frac{7\pi}{24} + k\pi / k \in Z \right\} :$$

$$\sin\left(3x - \frac{2\pi}{3}\right) = \frac{\sqrt{3}}{2} = \sin\left(\frac{\pi}{3}\right) :$$

$$2 \sin\left(3x - \frac{2\pi}{3}\right) = \sqrt{3}$$

$$3x - \frac{2\pi}{3} = \pi - \frac{\pi}{3} + 2k\pi / k \in Z \quad 3x - \frac{2\pi}{3} = \frac{\pi}{3} + 2k\pi / k \in Z :$$

$$3x = \frac{2\pi}{3} + \frac{2\pi}{3} + 2k\pi / k \in Z \quad 3x = \frac{\pi}{3} + \frac{2\pi}{3} + 2k\pi / k \in Z :$$

$$3x = \frac{4\pi}{3} + 2k\pi / k \in Z \quad 3x = \pi + 2k\pi / k \in Z :$$

$$x = \frac{4\pi}{9} + \frac{2k\pi}{3} / k \in Z \quad x = \frac{\pi}{3} + \frac{2k\pi}{3} / k \in Z :$$

$$S = \left\{ \frac{\pi}{3} + \frac{2k\pi}{3} / k \in Z \right\} \cup \left\{ \frac{4\pi}{9} + \frac{2k\pi}{3} / k \in Z \right\} :$$

$$a = \pi - b + 2k\pi / k \in Z \quad a = b + 2k\pi / k \in Z : \quad \sin(a) = \sin(b) : \quad \square$$

$$a = -b + 2k\pi / k \in Z \quad a = b + 2k\pi / k \in Z : \quad \cos(a) = \cos(b)$$

$$\sin\left(5x - \frac{\pi}{3}\right) = \sin\left(x + \frac{\pi}{3}\right) :$$

$$5x - \frac{\pi}{3} = \pi - \left(x + \frac{\pi}{3}\right) + 2k\pi / k \in \mathbb{Z} \quad 5x - \frac{\pi}{3} = x + \frac{\pi}{3} + 2k\pi / k \in \mathbb{Z}$$

$$5x - \frac{\pi}{3} = \pi - x - \frac{\pi}{3} + 2k\pi / k \in \mathbb{Z} \quad 5x - x = \frac{\pi}{3} + \frac{\pi}{3} + 2k\pi / k \in \mathbb{Z}$$

$$6x = \pi + 2k\pi / k \in \mathbb{Z} \quad 4x = \frac{2\pi}{3} + 2k\pi / k \in \mathbb{Z}$$

$$x = \frac{\pi}{6} + \frac{k\pi}{3} / k \in \mathbb{Z} \quad x = \frac{\pi}{6} + \frac{k\pi}{2} / k \in \mathbb{Z}$$

$$S = \left\{ \frac{\pi}{6} + \frac{k\pi}{2} / k \in \mathbb{Z} \right\} \cup \left\{ \frac{\pi}{6} + \frac{k\pi}{3} / k \in \mathbb{Z} \right\} \quad :$$

$$\cos\left(\frac{\pi}{2} - x\right) = \cos x : \quad \sin x = \cos x :$$

$$\frac{\pi}{2} - x = -x + 2k\pi / k \in \mathbb{Z} \quad \frac{\pi}{2} - x = x + 2k\pi / k \in \mathbb{Z}$$

$$\frac{\pi}{2} = 2k\pi / k \in \mathbb{Z} \quad -2x = \frac{-\pi}{2} + 2k\pi / k \in \mathbb{Z}$$

$$\left(\frac{1}{2} = 0,5 \text{ متساوية غير ممكنة لأن } 2k \text{ عدد صحيح}\right) \quad \frac{1}{2} = 2k / k \in \mathbb{Z} \quad x = \frac{\pi}{4} - k\pi / k \in \mathbb{Z}$$

$$S = \left\{ \frac{\pi}{4} - k\pi / k \in \mathbb{Z} \right\} \quad :$$

$$\cos(3x) = 0 \quad \sin x - 1 = 0 : \quad \cos(3x) . (\sin x - 1) = 0 :$$

$$3x = 2k\pi / k \in \mathbb{Z} \quad x = \frac{\pi}{2} + 2k\pi / k \in \mathbb{Z} : \quad \cos(3x) = 0 \quad \sin x = 1 :$$

$$x = \frac{2k\pi}{3} / k \in \mathbb{Z} \quad x = \frac{\pi}{2} + 2k\pi / k \in \mathbb{Z} :$$

$$S = \left\{ \frac{\pi}{2} + 2k\pi / k \in \mathbb{Z} \right\} \cup \left\{ \frac{2k\pi}{3} / k \in \mathbb{Z} \right\} \quad :$$

القيم 0 1 - 1- خاصية :

$$\sin x \left(2 + \frac{1}{\cos x} \right) = 0 : \quad 2 \sin x + \frac{\sin x}{\cos x} = 0 : \quad 2 \sin x + \tan x = 0 :$$

$$\frac{1}{\cos x} = -2 \quad \sin x = 0 : \quad 2 + \frac{1}{\cos x} = 0 \quad \sin x = 0 :$$

$$\cos x = \cos \left(\frac{2\pi}{3} \right) \quad \sin x = 0 : \quad \cos x = \frac{-1}{2} \quad \sin x = 0 :$$

$$x = -\frac{2\pi}{3} + 2k\pi / k \in \mathbb{Z} \quad x = \frac{2\pi}{3} + 2k\pi / k \in \mathbb{Z} \quad x = k\pi / k \in \mathbb{Z}$$

$$S = \left\{ k\pi / k \in \mathbb{Z} \right\} \cup \left\{ \frac{2\pi}{3} + 2k\pi / k \in \mathbb{Z} \right\} \cup \left\{ -\frac{2\pi}{3} + 2k\pi / k \in \mathbb{Z} \right\} : \quad$$

$$\cos \left(\frac{\pi}{2} - x \right) = \cos x : \quad \sin x = \cos x :$$

$$\frac{\pi}{2} - x = -x + 2k\pi / k \in \mathbb{Z} \quad \frac{\pi}{2} - x = x + 2k\pi / k \in \mathbb{Z}$$

$$\frac{\pi}{2} = 2k\pi / k \in \mathbb{Z} \quad -2x = \frac{-\pi}{2} + 2k\pi / k \in \mathbb{Z}$$

$$\left(\frac{1}{2} = 0,5 \text{ متساوية غير ممكنة لأن } 2k \text{ عدد صحيح و } \frac{1}{2} = 2k / k \in \mathbb{Z} \right) \quad x = \frac{\pi}{4} - k\pi / k \in \mathbb{Z}$$

$$S = \left\{ \frac{\pi}{4} - k\pi / k \in \mathbb{Z} \right\} : \quad$$

$$\cos \left(\frac{\pi}{2} - \left(x + \frac{\pi}{6} \right) \right) = \cos \left(2x + \frac{\pi}{3} \right) : \quad \sin \left(x + \frac{\pi}{6} \right) = \cos \left(2x + \frac{\pi}{3} \right) :$$

$$\cos \left(\frac{\pi}{3} - x \right) = \cos \left(2x + \frac{\pi}{3} \right) : \quad \cos \left(\frac{\pi}{2} - x - \frac{\pi}{6} \right) = \cos \left(2x + \frac{\pi}{3} \right) :$$

$$\frac{\pi}{3} - x = -\left(2x + \frac{\pi}{3} \right) + 2k\pi / k \in \mathbb{Z} \quad \frac{\pi}{3} - x = 2x + \frac{\pi}{3} + 2k\pi / k \in \mathbb{Z} :$$

$$\frac{\pi}{3} - x = -2x - \frac{\pi}{3} + 2k\pi / k \in \mathbb{Z} \quad -3x = 2k\pi / k \in \mathbb{Z} :$$

$$x = -\frac{2\pi}{3} + 2k\pi / k \in \mathbb{Z} \quad x = \frac{-2k\pi}{3} / k \in \mathbb{Z} :$$

$$S = \left\{ -\frac{2k\pi}{3} / k \in \mathbb{Z} \right\} \cup \left\{ -\frac{2\pi}{3} + 2k\pi / k \in \mathbb{Z} \right\} : \quad$$

: نطبق القواعد السابقة حتى وإن كانت التعبيرات داخل النسب المثلثية مركبة.

$$\sin x + 2 = 0 \quad \sin x - 2 = 0 \quad : \quad (\sin x - 2)(\sin x + 2) = 0 \quad : \quad \sin^2 x - 4 = 0 \quad :$$

$-1 \leq \sin x \leq 1$ وهذا غير ممكن في الحالتين لأننا نعلم أن: $\sin x = -2 \quad \sin x = 2$:

$$S = \emptyset \quad :$$

$$(\sqrt{3} \tan x - 1)(\sqrt{3} \tan x + 1) = 0 \quad : \quad 3 \tan^2(x) - 1 = 0 \quad :$$

$$\tan x = \frac{-\sqrt{3}}{3} \quad \tan x = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

$$x = -\frac{\pi}{6} + k\pi / k \in Z \quad x = \frac{\pi}{6} + k\pi / k \in Z \quad : \quad \tan x = \tan\left(-\frac{\pi}{6}\right) \quad \tan x = \tan\left(\frac{\pi}{6}\right)$$

$$S = \left\{ \frac{\pi}{6} + k\pi / k \in Z \right\} \cup \left\{ -\frac{\pi}{6} + k\pi / k \in Z \right\} \quad :$$

$$(\sqrt{2} \cos x - 1)(\sqrt{2} \cos x + 1) = 0 \quad : \quad 4 \cos^2 x - 1 = 0 \quad :$$

$$\cos x = \cos\left(\frac{3\pi}{4}\right) \quad \cos x = \cos\left(\frac{\pi}{4}\right) : \quad \cos x = \frac{-\sqrt{2}}{2} \quad \cos x = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} :$$

:

$$x = -\frac{3\pi}{4} + 2k\pi / k \in Z \quad x = \frac{3\pi}{4} + 2k\pi / k \in Z \quad x = -\frac{\pi}{4} + 2k\pi / k \in Z \quad x = \frac{\pi}{4} + 2k\pi / k \in Z$$

$$S = \left\{ \frac{\pi}{4} + 2k\pi / k \in Z \right\} \cup \left\{ -\frac{\pi}{4} + 2k\pi / k \in Z \right\} \cup \left\{ \frac{3\pi}{4} + 2k\pi / k \in Z \right\} \cup \left\{ -\frac{3\pi}{4} + 2k\pi / k \in Z \right\} :$$