السنة 1 بكالوريا علوم تجريبية	الحساب المثلثي	سلسلة 3

تمرين 1 : حل في IR المعادلات التالية:

$\cos x - \sin x = \sqrt{2}$	$\cos x - \sqrt{3}\sin x = 1$	$\sqrt{3}\cos x - \sin x = \sqrt{2}$
$\cos\frac{x}{2} - \sin\frac{x}{2} = -1$	$\sin x + \cos x = \sqrt{2}$	$\sin x + \cos x = 1$

تمرين 2 : نعتبر الدالة : $f(x) = \dfrac{\cos x + \sin x}{\cos x - \sin x}$

1) حل في IR المعادلة : $\cos x - \sin x = 0$

2) حدد Df

3) بين أن : $f(x) = \dfrac{1 + \sin 2x}{\cos 2x} \quad \forall x \in Df$

4) حل في IR المعادلة : $f(x) - \sqrt{3} = 0$

تمرين 3 : ليكن x عددا حقيقيا، نعتبر التعبير : $A(x) = \sqrt{3}\cos\left(2x - \frac{\pi}{3}\right) - \sin\left(2x - \frac{\pi}{3}\right)$

1) بين أن : $A(x) = 2\cos\left(2x - \frac{\pi}{6}\right)$

2) حل في المجال $\left]-\pi ; \pi\right]$ المعادلة : $A(x) = 1$

تمرين 4 : ليكن x عددا حقيقيا، نعتبر التعبير : $A(x) = -2\cos^2(x) + \sqrt{3}\sin(2x) + 2$

1) بين أن: $A(x) = 4\sin(x)\left(\frac{\sqrt{3}}{2}\cos(x) + \frac{1}{2}\sin(x)\right)$

2) حل في IR المعادلة : $A(x) = 0$

سلسلة 3	الحساب المثلثي حلول مقترحة	السنة 1 بكالوريا علوم تجريبية

تمرين 1 : المعادلات الموجودة بالتمرين كلها على شكل: $a\cos x + b\sin x = c$ ، حيث نعمل و نختزل بـ $r = \sqrt{a^2 + b^2}$

لنحل المعادلة: $\cos x - \sin x = \sqrt{2}$

لدينا:

$$\frac{1}{\sqrt{2}}\cos x - \frac{1}{\sqrt{2}}\sin x = 1$$
$$\cos\left(\frac{\pi}{4}\right)\cos x - \sin\left(\frac{\pi}{4}\right)\sin x = 1$$
$$\cos\left(\frac{\pi}{4} + x\right) = 1$$
$$x + \frac{\pi}{4} = 2k\pi \;/\; k \in Z$$
$$x = \frac{-\pi}{4} + 2k\pi \;/\; k \in Z$$

بالتالي: $S = \left\{\frac{-\pi}{4} + 2k\pi \;/\; k \in Z\right\}$

لنحل المعادلة: $\cos x - \sqrt{3}\sin x = 1$

$$\frac{1}{2}\cos x - \frac{\sqrt{3}}{2}\sin x = \frac{1}{2}$$
$$\cos\left(\frac{\pi}{3}\right)\cos x - \sin\left(\frac{\pi}{3}\right)\sin x = \frac{1}{2}$$
$$\cos\left(\frac{\pi}{3} + x\right) = \frac{1}{2} = \cos\left(\frac{\pi}{3}\right)$$
$$x + \frac{\pi}{3} = \frac{\pi}{3} + 2k\pi \;/\; k \in Z \;\; ou \;\; x + \frac{\pi}{3} = \frac{-\pi}{3} + 2k\pi \;/\; k \in Z$$
$$x = 2k\pi \;/\; k \in Z \;\; ou \;\; x = \frac{-2\pi}{3} + 2k\pi \;/\; k \in Z$$

بالتالي: $S = \{2k\pi \;/\; k \in Z\} \cup \left\{\frac{-2\pi}{3} + 2k\pi \;/\; k \in Z\right\}$

لنحل المعادلة: $\sqrt{3}\cos x - \sin x = \sqrt{2}$

لدينا:

$$\frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\sin x = \frac{\sqrt{2}}{2}$$
$$\cos\left(\frac{\pi}{4}\right)\cos x - \sin\left(\frac{\pi}{4}\right)\sin x = \frac{\sqrt{2}}{2}$$
$$\cos\left(\frac{\pi}{4} + x\right) = \cos\left(\frac{\pi}{4}\right)$$
$$x + \frac{\pi}{4} = \frac{\pi}{3} + 2k\pi \;/\; k \in Z \;\; ou \;\; x + \frac{\pi}{4} = \frac{-\pi}{3} + 2k\pi \;/\; k \in Z$$
$$x = \frac{\pi}{3} - \frac{\pi}{4} + 2k\pi \;/\; k \in Z \;\; ou \;\; x = \frac{-\pi}{3} - \frac{\pi}{4} + 2k\pi \;/\; k \in Z$$
$$x = \frac{\pi}{12} + 2k\pi \;/\; k \in Z \;\; ou \;\; x = \frac{-7\pi}{12} + 2k\pi \;/\; k \in Z$$

بالتالي: $S = \left\{\frac{\pi}{12} + 2k\pi \;/\; k \in Z\right\} \cup \left\{\frac{-7\pi}{12} + 2k\pi \;/\; k \in Z\right\}$

لنحل المعادلة: $\cos\frac{x}{2}-\sin\frac{x}{2}=-1$

لدينا:

$$\frac{1}{\sqrt{2}}\cos\frac{x}{2}-\frac{1}{\sqrt{2}}\sin\frac{x}{2}=-1$$

$$\cos\left(\frac{\pi}{4}\right)\cos\frac{x}{2}-\sin\left(\frac{\pi}{4}\right)\sin\frac{x}{2}=-1$$

$$\cos\left(\frac{\pi}{4}+\frac{x}{2}\right)=-1$$

$$\frac{x}{2}+\frac{\pi}{4}=\pi+k\pi\ /\ k\in Z$$

$$\frac{x}{2}=\frac{3\pi}{4}+k\pi\ /\ k\in Z$$

$$x=\frac{3\pi}{2}+2k\pi\ /\ k\in Z$$

بالتالي: $S=\left\{\frac{3\pi}{2}+2k\pi\ /\ k\in Z\right\}$

لنحل المعادلة: $\sin x+\cos x=\sqrt{2}$

لدينا:

$$\frac{1}{\sqrt{2}}\cos x+\frac{1}{\sqrt{2}}\sin x=1$$

$$\cos\left(\frac{\pi}{4}\right)\cos x+\sin\left(\frac{\pi}{4}\right)\sin x=1$$

$$\cos\left(\frac{\pi}{4}-x\right)=1$$

$$x-\frac{\pi}{4}=2k\pi\ /\ k\in Z$$

$$x=\frac{\pi}{4}+2k\pi\ /\ k\in Z$$

بالتالي: $S=\left\{\frac{\pi}{4}+2k\pi\ /\ k\in Z\right\}$

لنحل المعادلة: $\sin x+\cos x=1$

لدينا:

$$\frac{1}{\sqrt{2}}\cos x+\frac{1}{\sqrt{2}}\sin x=\frac{1}{\sqrt{2}}$$

$$\cos\left(\frac{\pi}{4}\right)\cos x+\sin\left(\frac{\pi}{4}\right)\sin x=\frac{\sqrt{2}}{2}$$

$$\cos\left(\frac{\pi}{4}-x\right)=\frac{\sqrt{2}}{2}=\cos\left(\frac{\pi}{4}\right)$$

$$\frac{\pi}{4}-x=\frac{\pi}{4}+2k\pi\ /\ k\in Z\ ou\ \frac{\pi}{4}-x=\frac{-\pi}{4}+2k\pi\ /\ k\in Z$$

$$x=-2k\pi\ /\ k\in Z\ ou\ x=\frac{\pi}{2}+2k\pi\ /\ k\in Z$$

بالتالي: $S=\{-2k\pi\ /\ k\in Z\}\cup\left\{\frac{\pi}{2}+2k\pi\ /\ k\in Z\right\}$

$\cos a=\cos b \Leftrightarrow a=b+2k\pi\ \ ou\ a=-b+2k\pi\ \ /\ k\in Z$

$\cos a=1 \Leftrightarrow a=2k\pi\ /\ k\in Z$; $\cos a=-1 \Leftrightarrow a=\pi+2k\pi\ /\ k\in Z$; $\cos a=0 \Leftrightarrow a=\frac{\pi}{2}+k\pi\ /\ k\in Z$

$\sin a=\sin b \Leftrightarrow a=b+2k\pi\ /\ k\in Z\ \ ou\ \ a=\pi-b+2k\pi\ /\ k\in Z$

$\sin a=1 \Leftrightarrow a=\frac{\pi}{2}+2k\pi\ /\ k\in Z$; $\sin a=-1 \Leftrightarrow a=\frac{-\pi}{2}+2k\pi\ /\ k\in Z$; $\sin a=0 \Leftrightarrow a=k\pi\ /\ k\in Z$

تمرين 2 : $f(x)=\dfrac{\cos x+\sin x}{\cos x-\sin x}$

1	لنحل المعادلة: $\sin x-\cos x=0$ لدينا : $\dfrac{1}{\sqrt{2}}\cos x-\dfrac{1}{\sqrt{2}}\sin x=0$ $\cos\left(\dfrac{\pi}{4}\right)\cos x-\sin\left(\dfrac{\pi}{4}\right)\sin x=0$ $\cos\left(\dfrac{\pi}{4}+x\right)=0$ $\dfrac{\pi}{4}+x=\dfrac{\pi}{2}+k\pi\ /\ k\in Z$ $x=\dfrac{\pi}{4}+k\pi\ /\ k\in Z$ بالتالي : $S=\left\{\dfrac{\pi}{4}+k\pi\ /\ k\in Z\right\}$
2	لدينا : $f(x)=\dfrac{\cos x+\sin x}{\cos x-\sin x}$ إذن حسب السؤال السابق : $Df=\{x\in IR/\cos x-\sin x\neq 0\}=IR_{-\left\{\frac{\pi}{4}+k\pi\ /\ k\in Z\right\}}$
3	لدينا: $f(x)=\dfrac{(\cos x+\sin x)}{(\cos x-\sin x)}=\dfrac{(\cos x+\sin x)(\cos x+\sin x)}{(\cos x-\sin x)(\cos x+\sin x)}$ $f(x)=\dfrac{(\cos x+\sin x)^2}{\cos^2 x-\sin^2 x}=\dfrac{\cos^2 x+\sin^2 x+2\sin x\cos x}{\cos x\cos x-\sin x\sin x}=\dfrac{1+\sin 2x}{\cos(x+x)}=\dfrac{1+\sin 2x}{\cos 2x}$
4	لنحل المعادلة: $f(x)-\sqrt{3}=0$ ، لدينا : $f(x)=\sqrt{3}\Leftrightarrow\dfrac{1+\sin 2x}{\cos 2x}=\sqrt{3}\Leftrightarrow 1+\sin 2x=\sqrt{3}\cos 2x\Leftrightarrow\sqrt{3}\cos 2x-\sin 2x=1$ $\Leftrightarrow\dfrac{\sqrt{3}}{2}\cos 2x-\dfrac{1}{2}\sin 2x=\dfrac{1}{2}\Leftrightarrow\cos\left(\dfrac{\pi}{6}\right)\cos 2x-\sin\left(\dfrac{\pi}{6}\right)\sin 2x=\dfrac{1}{2}$ $\Leftrightarrow\cos\left(\dfrac{\pi}{6}+2x\right)=\cos\left(\dfrac{\pi}{3}\right)$ $\Leftrightarrow\dfrac{\pi}{6}+2x=\dfrac{\pi}{3}+2k\pi\ /\ k\in Z\ ou\ \dfrac{\pi}{6}+2x=\dfrac{-\pi}{3}+2k\pi\ /\ k\in Z$ $\Leftrightarrow 2x=\dfrac{\pi}{6}+2k\pi\ /\ k\in Z\ ou\ 2x=\dfrac{-\pi}{2}+2k\pi\ /\ k\in Z$ $f(x)=\sqrt{3}\Leftrightarrow x=\dfrac{\pi}{12}+k\pi\ /\ k\in Z\ ou\ x=\dfrac{-\pi}{4}+k\pi\ /\ k\in Z$ بالتالي: $S=\left\{\dfrac{\pi}{12}+k\pi\ /\ k\in Z\right\}\cup\left\{\dfrac{-\pi}{4}+k\pi\ /\ k\in Z\right\}$

تمرين 3 : $A(x)=\sqrt{3}\cos\left(2x-\dfrac{\pi}{3}\right)-\sin\left(2x-\dfrac{\pi}{3}\right)$

1	$A(x)=\sqrt{3}\cos\left(2x-\dfrac{\pi}{3}\right)-\sin\left(2x-\dfrac{\pi}{3}\right)=2\left(\dfrac{\sqrt{3}}{2}\cos\left(2x-\dfrac{\pi}{3}\right)-\dfrac{1}{2}\sin\left(2x-\dfrac{\pi}{3}\right)\right)$ $=2\left(\cos\left(\dfrac{\pi}{6}\right)\cos\left(2x-\dfrac{\pi}{3}\right)-\sin\left(\dfrac{\pi}{6}\right)\sin\left(2x-\dfrac{\pi}{3}\right)\right)$ $A(x)=2\cos\left(\dfrac{\pi}{6}+2x-\dfrac{\pi}{3}\right)=2\cos\left(2x-\dfrac{\pi}{6}\right)$

$$A(x)=1 \Leftrightarrow 2\cos\left(2x-\frac{\pi}{6}\right)=1 \Leftrightarrow \cos\left(2x-\frac{\pi}{6}\right)=\frac{1}{2}=\cos\left(\frac{\pi}{3}\right)$$

$$\Leftrightarrow 2x-\frac{\pi}{6}=\frac{\pi}{3}+2k\pi \;/\; k\in Z \quad ou \quad 2x-\frac{\pi}{6}=\frac{-\pi}{3}+2k\pi \;/\; k\in Z$$

$$\Leftrightarrow 2x=\frac{\pi}{2}+2k\pi \;/\; k\in Z \quad ou \quad 2x=\frac{-\pi}{6}+2k\pi \;/\; k\in Z$$

$$A(x)=1 \Leftrightarrow x=\frac{\pi}{4}+k\pi \;/\; k\in Z \quad ou \quad x=\frac{-\pi}{12}+k\pi \;/\; k\in Z$$

لتحديد الحلول الموجودة في المجال $]-\pi;\pi]$ نؤطر العدد النسبي k و نحدد قيمته:

$$\frac{\pi}{4}+k\pi\in]-\pi;\pi] \Leftrightarrow -\pi<\frac{\pi}{4}+k\pi\le\pi \Leftrightarrow -4\pi<\pi+4k\pi\le4\pi \Leftrightarrow -5\pi<4k\pi\le3\pi$$

$$\Leftrightarrow -5<4k\le3 \quad \Leftrightarrow \frac{-5}{4}<k\le\frac{3}{4} \Leftrightarrow \quad k=-1 \; ou \; k=0$$ 2

$$\frac{\pi}{4}+k\pi\in]-\pi;\pi] \Leftrightarrow \quad x=\frac{\pi}{4}-\pi=\frac{-3\pi}{4} \quad ou \quad x=\frac{\pi}{4}$$

$$\frac{-\pi}{12}+k\pi\in]-\pi;\pi] \Leftrightarrow -\pi<\frac{-\pi}{12}+k\pi\le\pi \Leftrightarrow -12\pi<-\pi+12k\pi\le12\pi \Leftrightarrow -11\pi<12k\pi\le13\pi$$

و:

$$\Leftrightarrow -11<12k\le13 \quad \Leftrightarrow \frac{-11}{12}<k\le\frac{13}{12} \Leftrightarrow \quad k=0 \; ou \; k=1$$

$$\frac{-\pi}{12}+k\pi\in]-\pi;\pi] \Leftrightarrow \quad x=\frac{-\pi}{12} \quad ou \quad x=\frac{-\pi}{12}+\pi=\frac{11\pi}{12}$$

بالتالي: $S=\left\{\frac{\pi}{4};\frac{-3\pi}{4};\frac{-\pi}{12};\frac{11\pi}{12}\right\}$

تمرين 4 : $A(x)=-2\cos^2(x)+\sqrt{3}\sin(2x)+2$

1

لدينا:

$$A(x)=-2\cos^2(x)+\sqrt{3}\sin(2x)+2=2(1-\cos^2 x)+\sqrt{3}\times2\sin x\cos x$$

$$=2\sin^2 x+2\sqrt{3}\sin x\cos x=4\sin x\left(\frac{1}{2}\sin x+\frac{\sqrt{3}}{2}\cos x\right)$$

$$A(x)=4\sin(x)\left(\frac{\sqrt{3}}{2}\cos(x)+\frac{1}{2}\sin(x)\right)$$

2

$$A(x)=0 \quad \Leftrightarrow \quad 4\sin(x)\left(\frac{\sqrt{3}}{2}\cos(x)+\frac{1}{2}\sin(x)\right)=0$$

$$\Leftrightarrow \quad \sin(x)=0 \quad ou \quad \left(\frac{\sqrt{3}}{2}\cos(x)+\frac{1}{2}\sin(x)\right)=0$$

$$\Leftrightarrow \quad x=k\pi \;/\; k\in Z \quad ou \quad \cos\left(\frac{\pi}{6}\right)\cos x+\sin\left(\frac{\pi}{6}\right)\sin x=0$$

$$\Leftrightarrow \quad x=k\pi \;/\; k\in Z \quad ou \quad \cos\left(x-\frac{\pi}{6}\right)=0$$

$$\Leftrightarrow \quad x=k\pi \;/\; k\in Z \quad ou \quad x-\frac{\pi}{6}=\frac{\pi}{2}+k\pi \;/\; k\in Z$$

$$A(x)=0 \quad \Leftrightarrow \quad x=k\pi \;/\; k\in Z \quad ou \quad x=\frac{2\pi}{3}+k\pi \;/\; k\in Z$$

بالتالي: $S=\{k\pi \;/\; k\in Z\}\cup\left\{\frac{2\pi}{3}+k\pi \;/\; k\in Z\right\}$