الامتحان الوطني الموحد للبكالوريا
الدورة العادية 2024
- الموضوع -

SSSSSSSSSSSSSSSSSSSS-SSSS | NS 24E

المملكة المغربية
وزارة التربية الوطنية والتعليم الأولي والرياضة
ⵜⴰⴳⵍⴷⵉⵜ ⵏ ⵍⵎⵖⵔⵉⴱ
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المركز الوطني للتقويم والامتحانات

4h	مدة الإنجاز	الرياضيات	المادة
9	المعامل	شعبة العلوم الرياضية (أ) و (ب) (خيار انجليزية)	الشعبة المسلك

- The duration of the exam is four hours

-The exam consists of 5 independent exercises

- The exercises can be treated in the order chosen by the candidate

- Exercise1 concerns analysis ……….……………..…(7.5 pts)

- Exercise2 concerns analysis ……….……………..…(2.5 pts)

- Exercise3 concerns complex numbers.............…(3.5 pts)

- Exercise4 concerns algebra ………………........(3.5 pts)

- Exercise5 concerns arithmetic ………………........(3 pts)

The use of the calculator is not allowed

The use of red color on sheet paper is not permitted

EXERCISE1 :(7.5 points)

Let f be the numerical function defined on the interval $[1,+\infty[$ by:

$$f(1)=\frac{1}{2} \text{ and for all } x\in]1,+\infty[\ , \quad f(x)=\frac{\ln(x)}{x^2-1}$$

And let (C) be its curve in an orthonormal coordinate system $(O,\vec{i},\vec{j})$

0.5 1- Show that f is continuous on the right at the point 1

0.5 2- Calculate $\lim_{x\to+\infty} f(x)$, then interpret graphiqually the obtained result .

3- a) Let $x\in]1,+\infty[$.

0.25 By putting: $t=(x-1)^2$, verify that: $\frac{1-x+\ln(x)}{(x-1)^2}=\frac{-\sqrt{t}+\ln(1+\sqrt{t})}{t}$

0.5 b) Show that $(\forall t\in]0,+\infty[)$, $-\frac{1}{2}<\frac{-\sqrt{t}+\ln\left(1+\sqrt{t}\right)}{t}<\frac{-1}{2(1+\sqrt{t})}$

(you can use the Mean Value Theorem on the interval $[0;t]$)

0.25 c) Deduce that: $\lim_{x\to1^+}\frac{1-x+\ln(x)}{(x-1)^2}=-\frac{1}{2}$

0.5 4- a) Show that: $\forall x\in]1,+\infty[$, $\frac{f(x)-\frac{1}{2}}{x-1}=-\frac{\ln(x)}{x-1}\times\frac{1}{2(x+1)}+\frac{\ln(x)-x+1}{2(x-1)^2}$

0.5 b) Deduce that f is differentiable on the right at the point 1, then interpret graphically the obtained result.

5- For all $x\in[1,+\infty[$ we put $I(x)=\int_1^x\frac{t^2-1}{t^3}dt$ and $J(x)=\int_1^x\frac{t^2-1}{t^2}dt$

0.5 a) Show that: $\forall x\in[1,+\infty[,\ 0\le I(x)\le J(x)$

0.5 b) Show that for all $x\in[1,+\infty[$, $I(x)=\ln(x)-\frac{x^2-1}{2x^2}$ and $J(x)=\frac{(x-1)^2}{x}$

0.5 c) Show that: $\forall x\in]1,+\infty[,\ f'(x)=\frac{-2}{(x+1)^2}\times\frac{I(x)}{J(x)}$

0.5 d) Deduce that: $\forall x\in]1,+\infty[,\ -\frac{1}{2}\le f'(x)\le 0$

0.25 6- a) Set up the table of variation of the function f

0.5 b) Plot the curve (C) (We take $\|\vec{i}\|=1\text{cm}$ and $\|\vec{j}\|=2\text{cm}$)

0.5 7- Show that the equation $f(x)=x-1$ admits a unique solution a in $]1,2[$

8- Let $(a_n)_{n\in\mathbb{N}}$ be the numerical sequence defined by:

$$a_0\in[1,+\infty[\quad \text{and for all } n\in\mathbb{N}, \qquad a_{n+1}=1+f(a_n)$$

0.5 a) Show that: $(\forall n \in \mathbb{N}),\ |a_{n+1}-a| \leq \frac{1}{2}|a_n - a|$

0.5 b) Show by induction: $(\forall n \in \mathbb{N}),\ |a_n - a| \leq \left(\frac{1}{2}\right)^n |a_0 - a|$

0.25 c) Deduce that the sequence $(a_n)_{n\in\mathbb{N}}$ is convergent.

EXERCISE2 :(2.5 points)

Let F be the numerical function defined on the interval $[0;1]$ by: $F(x) = \int_0^x e^{t^2}\,dt$

0.5 1-a) Show that F is continuous and strictly increasing on $[0;1]$

0.5 b) Deduce that F is a bijection from $[0;1]$ to $[0;\beta]$ where $\beta = \int_0^1 e^{t^2}\,dt$

2- F^{-1} denotes the inverse bijection of F

For all $n \in \mathbb{N}^*$, we put: $S_n = \frac{1}{n}\sum_{k=1}^{k=n} F^{-1}\left(\frac{k}{n}\beta\right)$

0.5 a) Show that the sequence $(S_n)_{n\in\mathbb{N}^*}$ is convergent with limit $\ell = \frac{1}{\beta}\int_0^\beta F^{-1}(t)\,dt$

0.5 b) Show that $\ell = \frac{1}{\beta}\int_0^1 u\,e^{u^2}\,du$ (We can use the variable change $u = F^{-1}(t)$)

0.5 c) Deduce that: $\ell = \frac{e-1}{2\beta}$

EXERCISE3 : (3.5 points)

The complex plane is attached to the direct orthonormal coordinate system $(O,\vec{u},\vec{v})$

Consider in $\mathbb{C}$ the equation with variable z

$$(E_\alpha):\ z^2 - 2iz + \alpha = 0 \qquad \text{where } \alpha \in \mathbb{C}$$

Part I:

0.25 1-a) Show that the discriminant of the equation (E_α) is $\Delta = -4(1+\alpha)$

0.25 b) Determine the set of the values of α for which the equation (E_α) admits two distinct solutions in $\mathbb{C}$.

0.5 2- z_1 and z_2 denote the two solutions of the equation (E_α).

Determine $z_1 + z_2$ and $z_1 z_2$

Part II:

Let Ω , M_1 and M_2 be the points with respective affixes α , z_1 and z_2

1- Suppose $\alpha = m^2 - 2m$ where $m \in \mathbb{R}$

0.5 a) Determine z_1 et z_2 in terms of m

0.25 b) Deduce that the points O , M_1 and M_2 are collinear.

2-Suppose that the points O , M_1 and M_2 are non collinear.

0.25 a) Show that $\frac{z_1}{z_2}$ is a pure imaginary if and only if $Re\left(z_1\overline{z_2}\right)=0$

0.5 b) Show that: $\left|z_1-z_2\right|^2=\left|z_1+z_2\right|^2-4\,Re\left(z_1\overline{z_2}\right)$

0.25 c) Deduce that $\frac{z_1}{z_2}$ is a pure imaginary if and only if $\left|z_1-z_2\right|=2$

0.25 3-a) Show that: $\left(z_1-z_2\right)^2=\Delta$

0.5 b) Determine the set Γ of the points Ω for which the triangle OM_1M_2 is rectangle at O

EXERCISE4 : (3.5 points)

Recall that $(M_2(\mathbb{R}),+,\times)$ is a unitary no commutative ring with zero the matrix $O=\begin{pmatrix}0&0\\0&0\end{pmatrix}$ and unit the matrix $I=\begin{pmatrix}1&0\\0&1\end{pmatrix}$

Consider in $\mathbb{C}\times\mathbb{C}^*$ the law internal operational T defined by:

$$\forall\left((a,b),(c,d)\right)\in\left(\mathbb{C}\times\mathbb{C}^*\right)^2\ ;\ (a,b)T(c,d)=\left(a\overline{d}+c,\ bd\right)$$

($\overline{d}$ is the conjugate of the complex number d)

0.5 1-a) Verify that $(i,2)T(1,i)=(2,2i)$, then calculate $(1,i)T(i,2)$

0.25 b) Deduce that the law T is not commutative in $\mathbb{C}\times\mathbb{C}^*$

0.5 2- Show that T is associative in $\mathbb{C}\times\mathbb{C}^*$

0.25 3- Verify that $(0,1)$ is the neutral element of T in $\mathbb{C}\times\mathbb{C}^*$

0.5 4-a) Verify that $\forall(a,b)\in\mathbb{C}\times\mathbb{C}^*\ ;\ (a,b)T\left(-\frac{a}{\overline{b}},\frac{1}{b}\right)=(0,1)$

0.5 b) Show that $\left(\mathbb{C}\times\mathbb{C}^*,T\right)$ is a non-commutative group.

0.5 5-a) Show that $\mathbb{R}\times\mathbb{R}^*$ is stable for the law T

0.5 b) Show that $\mathbb{R}\times\mathbb{R}^*$ is a sub-group of the group $\left(\mathbb{C}\times\mathbb{C}^*,T\right)$

EXERCISE5 :(3 points)

Let p and q be distinct prime numbers and r be a natural number which is prime with p and with q

1 1- a) Show that p divides $r^{p-1}-1$ and that q divides $r^{q-1}-1$

0.5 b) Deduce that p and q divides $r^{(p-1)(q-1)}-1$

0.5 c) Show that pq divides $r^{(p-1)(q-1)}-1$

1 2- Solve in $\mathbb{Z}$ the equation $2024^{192}x \equiv 3 \quad [221]$ (We give : $221 = 13 \times 17$)

END