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الامتحان الوطني الموحد للبكالوريا
المسالك الدولية - خيار إنجليزية
الدورة الاستدراكية 2017
- الموضوع -
RS 22E

المملكة المغربية
وزارة التربية الوطنية والتكوين المهني والتعليم العالي والبحث العلمي
المركز الوطني للتقويم والامتحانات والتوجيه

3	مدة الإنجاز	الرياضيات	المادة
7	المعامل	مسلك العلوم الفيزيائية – خيار إنجليزية	الشعبة أو المسلك

GENERAL INSTRUCTIONS

- The use of non- programmable calculator is allowed ;
- The exercises can be treated in the preferred order by the candidate ;
- The use of red color when drafting solutions is to be avoided .

COMPONENTS OF THE EXAM

- The exam consists of four exercises and a problem , independent of each other according to the fields as follows :

Exercise 1	Geometry in space.	3 points
Exercise 2	Caculating probabilities.	3 points
Exercise 3	Complex numbers.	3 points
Exercise 4	Numerical sequences.	2.5 points
Problem	Study of numerical function and calculating integrals	8.5 points

Exercise 1 (3 points)

In the space referred to an orthonormal direct coordinate system $\left(O,\vec{i},\vec{j},\vec{k}\right)$,

we consider the sphere $\left(S\right)$ of equation $x^2+y^2+z^2-2x-2y-2z-1=0$ and the plane (P) of equation $y-z=0$

0.5 1) a) Show that the sphere $\left(S\right)$ has the center $\Omega\left(1,1,1\right)$ and the radius 2

0.5 b) Calculate $d\left(\Omega,(P)\right)$ and then deduce that the plane (P) intersects the sphere $\left(S\right)$ along a circle $\left(C\right)$

0.5 c) Determine the center and the radius of the circle $\left(C\right)$

2) Let (Δ) be the line passing through the point $A\left(1,-2,2\right)$ and perpendicular to the plane (P)

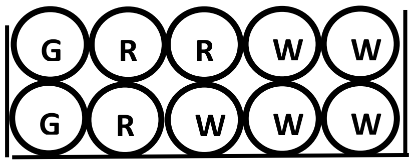
0.25 a) Show that $\vec{u}\left(0,1,-1\right)$ is a director vector of the line (Δ)

0.75 b) Show that $\left\|\overrightarrow{\Omega A}\wedge\vec{u}\right\|=\sqrt{2}\left\|\vec{u}\right\|$ and then deduce that the line (Δ) intersects the sphere $\left(S\right)$ at two points .

0.5 c) Determine the coordinates of each point of intersection of the line (Δ) and the sphere $\left(S\right)$

Exercise 2 (3 points)

An urn contains ten balls indistinguishable by touch :
five white balls , three red balls and two green balls.
(See the figure beside)



We draw , simultaneously and randomly, four balls from the urn.

1.5 1) Let A be the event :'' Among the four drawn balls, only one ball is green ''.
and B the event:''Among the four drawn balls, there are exactly three balls of the same color''

Show that $p\left(A\right)=\dfrac{8}{15}$ and that $p(B)=\dfrac{19}{70}$

2) Let X be the random variable that associates to each draw the number of green drawn balls.

0.5 a) Show that $p(X=2)=\dfrac{2}{15}$

1 b) Determine the law of the probability of the random variable X and show that the expected value $E\left(X\right)$ is equal to $\dfrac{4}{5}$

Exercise 3 (3 points)

0.75 1) Solve in the set of complex numbers $\mathbb{C}$ the equation $z^2 + 4z + 8 = 0$

2) In the complex plane referred to an orthonormal direct coordinate system $(O,\vec{u},\vec{v})$,
we consider the points A, B and C of respective affixes a, b and c such that
$a = -2 + 2i$, $b = 4 - 4i$ and $c = 4 + 8i$

0.5 a) Let z be the affix of a point M in the plane and z' the affix of the point M', image of M by the rotation R with center A and angle $-\frac{\pi}{2}$

Show that $z' = -iz - 4$

0.75 b) Verify that the point B is the image of the point C by the rotation R and deduce the nature of the triangle ABC

3) Let ω be the affix of the point Ω, midpoint of the segment $[BC]$

0.5 a) Show that $|c - \omega| = 6$

0.5 b) Show that the set of points M of affix z such that $|z - \omega| = 6$ is the circle circumscribed to the triangle ABC

Exercise 4 (2.5 points)

We consider the numerical sequence (u_n) defined by :

$u_0 = 17$ and $u_{n+1} = \frac{1}{4} u_n + 12$ for every natural number n

0.5 1) a) Show by induction that $u_n > 16$ for every natural number n

0.5 b) Show that the sequence (u_n) is decreasing and then deduce that the sequence (u_n) is convergent.

2) Let (v_n) be the numerical sequence such that $v_n = u_n - 16$ for every natural number n

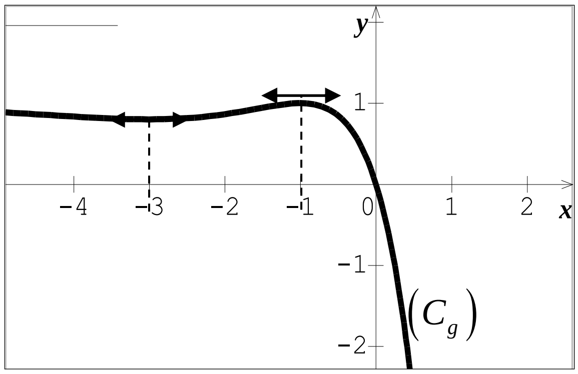
0.5 a) Show that (v_n) is a geometric sequence .

0.5 b) Deduce that $u_n = 16 + \left(\frac{1}{4}\right)^n$ for every natural number n, then determine the limit of the sequence (u_n)

0.5 c) Determine the smallest value of the natural number n for which $u_n < 16,0001$

Problem (8.5 points)

I- Let g be the numerical function defined on $\mathbb{R}$ by : $g(x)=1-(x+1)^2e^x$



0.25 **1) Verify that** $g(0)=0$

1 **2) From the representative curve** (C_g) **of the function** g **(see the figure beside)**
Show that :
$g(x)\geq 0$ **for every** x **in the interval** $]-\infty,0]$
and that $g(x)\leq 0$ **for every** x **in the interval** $[0,+\infty[$

II- We consider the numerical function f defined on $\mathbb{R}$ by : $f(x)=x+1-(x^2+1)e^x$
and let (C_f) **be the curve of** f **in an orthonormal coordinate system** $(O,\vec{i},\vec{j})$ **(unit:** $2\,cm$**)**

0.75 **1)a)Verify that** $f(x)=x+1-4\left(\frac{x}{2}e^{\frac{x}{2}}\right)^2-e^x$ **for every real** x **then deduce that** $\lim\limits_{x\to-\infty}f(x)=-\infty$

0.5 **b) Calculate** $\lim\limits_{x\to-\infty}\left[f(x)-(x+1)\right]$ **and then deduce that the line** (D) **of equation** $y=x+1$ **is asymptote to the curve** (C_f) **at** $-\infty$

0.25 **c) Show that the curve** (C_f) **is under the line** (D)

0.5 **2)a) Show that** $\lim\limits_{x\to+\infty}f(x)=-\infty$ **(we can write** $f(x)$ **in the form** $x\left[1+\frac{1}{x}-\left(x+\frac{1}{x}\right)e^x\right]$ **)**

0.25 **b) Show that the curve** (C_f) **has an asymptotic direction at** $+\infty$**, wich will be determined.**

0.75 **3)a) Show that** $f'(x)=g(x)$ **for every real** x

0.75 **b) Show that the function** f **is increasing on** $]-\infty\,,0]$ **and decreasing on** $[0,+\infty[$
then set up the table of variations of the function f **on** $\mathbb{R}$

0.75 **c) Show that the curve** (C_f) **admits two inflection points of abscissa** -3 **and** -1

1 **4) Sketch , the line** (D) **and the curve** (C_f) **in the same system coordinate** $(O,\vec{i},\vec{j})$**.**
(we take $f(-3)\approx-2{,}5$ **and** $f(-1)\approx-0{,}75$ **)**

0.5 **5)a) Verify that** $H:x\mapsto(x-1)e^x$ **is a primitive of the function** $h:x\mapsto xe^x$ **on** $\mathbb{R}$
then show that $\int_{-1}^{0}x\,e^x\,dx=\frac{2}{e}-1$

0.75 **b) Using an integration by parts, show that :** $\int_{-1}^{0}(x^2+1)e^x\,dx=3\left(1-\frac{2}{e}\right)$

0.5 **c) Calculate, in** cm^2**, the area enclosed between the curve** (C_f)**, the line** (D)**, the y-axis and the line of equation** $x=-1$

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On prendra en considération les différentes étapes menant à la solution .
On acceptera toute autre méthode correcte .

Exercice 1 (3 points)

1.5 **1)a) 0.25 pour le centre et 0.25 pour le rayon**

b) 0.25 pour $d\left(\Omega,(P)\right)=0$ **et 0.25 pour l'intersection est un cercle**

c) 0.25 pour le centre est Ω **et 0.25 pour le rayon est** 2

1.5 **2) a) 0.25** **b) 0.25 pour** $\overrightarrow{\Omega A}\wedge\vec{u}=2\vec{i}$ **, 0.25 pour l'égalité et 0.25 pour** $d\left(\Omega,(\Delta)\right)<2$

c) 0.25 pour le triplet $(1\,,1\,,-1)$ **et 0.25 pour le triplet** $(1,\,-1\,,1)$

Exercice 2 (3 points)

1.5 **1) 0.75 pour** $p(A)=\frac{8}{15}$ **et 0.75 pour** $p(B)=\frac{19}{70}$

1.5 **2) a) 0.5 pour** $p(X=2)=\frac{2}{15}$

b) 0.25 pour $p(X=1)=\frac{8}{15}$ **, 0.5 pour** $p(X=0)=\frac{1}{3}$ **et 0.25 pour** $E(X)=\frac{4}{5}$

Exercice 3 (3 points)

0.75 **1) 0.25 pour le calcul du discriminant et 0.25 pour chaque solution**
(on attribuera 0.75 pour toute autre méthode permettant de déterminer les deux solutions de l'équation)

1.25 **2) a) 0.25 pour l'écriture** $z'-a=e^{-i\frac{\pi}{2}}(z-a)$ **et 0.25 pour** $z'=-iz-4$

b) 0.25 pour $R(C)=B$ **et 0.5 pour le triangle est rectangle isocèle**

1 **3) a) 0.5**

b) 0.25 pour la traduction de l'écriture $|z-\omega|=6$ $(\Omega M=6)$ **et 0.25 pour l'ensemble des points .**

Exercice 4 (2.5 points)

1 **1) a) 0.5**

b) 0.25 pour la suite est décroissante et 0.25 pour la suite est convergente

1.5 **2) a) 0.5**

b) 0.25 pour la déduction et 0.25 pour la limite de (u_n) **est** 16

c) 0.5 (la plus petite valeur est 7)

Problème (8.5 points)

0.25	**I-1) 0.25 pour** $g(0)=0$
1	**2) 0.5 pour** $g(x)\geq 0$ **pour tout** x **de** $]-\infty,0]$ **et 0.5 pour** $g(x)\leq 0$ **pour tout** x **de** $[0,+\infty[$
1.5	**II-1) a) 0.25 pour la vérification et 0.5 pour la limite**
	b) 0.25 pour le calcul de la limite et 0.25 pour la déduction c)0.25
0.75	**2) a) 0.5 b) 0.25**
2.25	**3) a) 0.75**
	b) 0.25 pour f **croissante sur** $]-\infty,0]$ **et 0.25 pour** f **décroissante sur** $[0,+\infty[$
	et 0.25 pour le tableau de variations
	c) 0.75
1	**4) 1 (voir figure ci-dessous)**
1.75	**5) a) 0.25 pour la vérification et 0.25 pour le calcul**
	b) 0.5 pour la technique de l'intégration par parties et 0.25 pour le résultat
	c) 0.25 pour l'aire, en cm^2 **, est** $4\int_{-1}^{0}(x+1-f(x))dx$ **et 0.25 pour l'aire est** $12\left(1-\frac{2}{e}\right)cm^2$