

# Les limites

## Exercice (1)

Calculer les limites suivantes

$\lim_{x \rightarrow 1} \frac{x^2 - x}{x^2 - 1}$	$\lim_{x \rightarrow 2} \frac{x^2 - x - 2}{x^2 - 4}$	$\lim_{x \rightarrow -2} \frac{2x^2 + 3x - 2}{x^2 + x - 2}$
$\lim_{x \rightarrow -2} \frac{x^3 + 8}{x^2 + 3x + 2}$	$\lim_{x \rightarrow \frac{2}{3}} \frac{3x^2 - 5x + 2}{9x^2 - 4}$	$\lim_{x \rightarrow 3} \frac{\sqrt{2x + 3} - 3}{x - 3}$
$\lim_{x \rightarrow -1} \frac{2x^4 + 3x + 1}{x^3 + 2x^2 - 1}$	$\lim_{x \rightarrow 1} \frac{\sqrt{3x - 2} - 1}{\sqrt{x} - 1}$	$\lim_{x \rightarrow 4} \frac{x\sqrt{x} - 8}{x^2 - 16}$
$\lim_{x \rightarrow 5} \frac{5\sqrt{x} - x\sqrt{5}}{x\sqrt{x} - 5\sqrt{5}}$	$\lim_{x \rightarrow 2} \frac{\sqrt{x - 1} - \sqrt{3x - 2} + 1}{x - \sqrt{x - 1} - 1}$	$\lim_{x \rightarrow 1} \frac{\sqrt{x^2 + 3} - \sqrt{3x + 1}}{x - 3\sqrt{x} + 2}$

## Exercice (2)

Calculer les limites ci-dessous :

$$\lim_{x \rightarrow +\infty} 3x^2 - 5x + 2 \quad ; \quad \lim_{x \rightarrow -\infty} \frac{1}{2}x^3 - 4x^2 + 9 \quad ; \quad \lim_{x \rightarrow +\infty} (2x + 1)^2 - 5x^2 + 3$$

$$\lim_{x \rightarrow +\infty} \frac{2x^2 - x - 3}{(x - 1)} \quad ; \quad \lim_{x \rightarrow -\infty} \frac{3x^2 - 2x + 1}{(x - 2)^2} \quad ; \quad \lim_{x \rightarrow +\infty} \frac{x^3 - 2x}{x^2 - 2x}$$

$$\lim_{x \rightarrow +\infty} \frac{x + 1}{2x^2 - x - 1} \quad ; \quad \lim_{x \rightarrow +\infty} \frac{x^2 - 2x}{x + 1} - 2x \quad ; \quad \lim_{x \rightarrow +\infty} \frac{\sqrt{x + 1} - 2}{x + 1}$$

$$\lim_{x \rightarrow +\infty} \sqrt{x + 1} + 2x \quad ; \quad \lim_{x \rightarrow -\infty} \sqrt{2 - x} - x \quad ; \quad \lim_{x \rightarrow +\infty} \sqrt{2x - 1} - x$$

$$\lim_{x \rightarrow +\infty} \sqrt{x + 1} + 2x \quad ; \quad \lim_{x \rightarrow -\infty} \sqrt{x^2 + 1} + x \quad ; \quad \lim_{x \rightarrow +\infty} \sqrt{x^2 + x} - x$$

$$\lim_{x \rightarrow +\infty} \sqrt{x + 4} - \sqrt{x} \quad ; \quad \lim_{x \rightarrow +\infty} \frac{2\sqrt{x} - 3}{3\sqrt{x} + 2} \quad ; \quad \lim_{x \rightarrow +\infty} \frac{2x - 1}{\sqrt{x} + 2}$$

## Limites remarquables :

$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$	$\lim_{x \rightarrow 0} \frac{\tan x}{x} = 1$	$\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2} = \frac{1}{2}$
$\lim_{x \rightarrow 0} \frac{\sin(ax)}{x} = a$	$\lim_{x \rightarrow 0} \frac{\tan(ax)}{x} = a$	$\lim_{x \rightarrow 0} \frac{1 - \cos ax}{x^2} = \frac{a^2}{2}$

## Exercice (3)

Déterminer les limites suivantes :

$\lim_{\substack{x \rightarrow 1 \\ x > 1}} \frac{2x + 3}{x^2 - 1}$	$\lim_{\substack{x \rightarrow \frac{1}{2} \\ x > \frac{1}{2}}} \frac{4x - 3}{2x - 1}$	$\lim_{\substack{x \rightarrow -3 \\ x > -3}} \frac{2x + 1}{x^2 - 9}$
$\lim_{\substack{x \rightarrow -2 \\ x < -2}} \frac{x(x + 3)}{(x - 1)(x + 2)}$	$\lim_{\substack{x \rightarrow 0 \\ x < 0}} \left( \frac{1}{x} + \frac{x + 2}{x^2} \right)$	$\lim_{x < 1} \frac{2x^2 - x + 3}{x^2 - x}$
$\lim_{\substack{x \rightarrow 2 \\ x > 2}} \frac{\sqrt{x} - 2}{x^2 - 2x}$	$\lim_{\substack{x \rightarrow 0 \\ x < 0}} \frac{x + 2 - 2\sqrt{x + 1}}{x^2}$	$\lim_{\substack{x \rightarrow 3 \\ x > 3}} \frac{\sqrt{x - 3} + x^2 - 9}{x - 3}$

## Exercice (4)

Calculer les limites

$\lim_{x \rightarrow 0} \frac{x + 2 \sin x}{2 \tan 3x - x}$	$\lim_{x \rightarrow 0} \frac{4x - \sin 2x}{x + \tan 3x}$	$\lim_{x \rightarrow 0} \frac{1 - \cos 3x}{x^2}$
$\lim_{x \rightarrow 0} \frac{\cos x - 1}{x \sin 3x}$	$\lim_{x \rightarrow 0} \frac{\cos 3x - \cos x}{x^2}$	$\lim_{x \rightarrow 0} \frac{\cos x + \cos 3x - 2}{x^2}$
$\lim_{x \rightarrow 0} \frac{2 \sin x - \sin 2x}{x^3}$	$\lim_{x \rightarrow 1} \frac{\sin(\pi x)}{x - 1}$	$\lim_{x \rightarrow \pi} \frac{1 + \cos x}{\sin x}$
$\lim_{x \rightarrow 0} \frac{1 + \sin x}{1 - \cos x}$	$\lim_{x \rightarrow +\infty} \frac{\sin x}{x}$	$\lim_{x \rightarrow +\infty} \frac{3 - 2 \cos x}{x^2 + 1}$
$\lim_{x \rightarrow -\infty} \frac{2x - 3 \sin x}{2 \cos 3x - 5x}$	$\lim_{x \rightarrow 0} \frac{\cos ax - \cos bx}{x^2}$	$\lim_{\substack{x \rightarrow 0 \\ x < 0}} \frac{1 + \cos x}{\sin x}$

Limites et ordre

☞ si  $g(x) \leq f(x) \leq h(x)$  et  $\lim_{x \rightarrow \alpha} g(x) = \lim_{x \rightarrow \alpha} h(x) = l$  alors  $\lim_{x \rightarrow \alpha} f(x) = l$

☞ si  $|f(x) - l| \leq h(x)$  et  $\lim_{x \rightarrow \alpha} h(x) = 0$  alors  $\lim_{x \rightarrow \alpha} f(x) = l$

☞ si  $f(x) \geq g(x)$  et  $\lim_{x \rightarrow \alpha} g(x) = +\infty$  alors  $\lim_{x \rightarrow \alpha} f(x) = +\infty$

☞  $f(x) \leq g(x)$  et  $\lim_{x \rightarrow \alpha} g(x) = -\infty$  alors  $\lim_{x \rightarrow \alpha} f(x) = -\infty$