

Tronc Commun

Série 1 : Calcul trigonométrique

Exercice 1

- 1) Représenter sur le cercle trigonométrique les points d'abscisses curvilignes : $\frac{5\pi}{6}$; $\frac{-\pi}{3}$; $\frac{4\pi}{3}$; $\frac{55\pi}{3}$
- 2) a. Déterminer : $\cos\left(\frac{5\pi}{6}\right)$, $\cos\left(\frac{-\pi}{3}\right)$, $\cos\left(\frac{4\pi}{3}\right)$ et $\cos\left(\frac{55\pi}{3}\right)$
b. Déterminer : $\sin\left(\frac{5\pi}{6}\right)$, $\sin\left(\frac{-\pi}{3}\right)$, $\sin\left(\frac{4\pi}{3}\right)$ et $\sin\left(\frac{55\pi}{3}\right)$
c. Déterminer : $\tan\left(\frac{5\pi}{6}\right)$, $\tan\left(\frac{-\pi}{3}\right)$, $\tan\left(\frac{4\pi}{3}\right)$ et $\tan\left(\frac{55\pi}{3}\right)$

Exercice 2

Simplifier les expressions suivantes :

$$A = \cos^2\left(\frac{\pi}{11}\right) + \cos^2\left(\frac{3\pi}{11}\right) + \cos^2\left(\frac{5\pi}{22}\right) + \cos^2\left(\frac{9\pi}{22}\right)$$

$$B = \sin\left(\frac{\pi}{13}\right) + \sin\left(\frac{3\pi}{13}\right) + \sin\left(\frac{14\pi}{13}\right) + \sin\left(\frac{16\pi}{13}\right)$$

$$C = \tan\left(\frac{\pi}{7}\right) + \tan\left(\frac{3\pi}{7}\right) + \tan\left(\frac{4\pi}{7}\right) + \tan\left(\frac{6\pi}{7}\right)$$

Exercice 3

Simplifier les expressions suivantes :

$$A = \sin(x + 5\pi) + \sin\left(\frac{9\pi}{2} - x\right) + \sin(13\pi - x) + \sin\left(\frac{17\pi}{2} + x\right)$$

$$B = \cos(x + 9\pi) + \cos(13\pi - x) + \cos(x + 28\pi) + \cos(x)$$

$$C = \tan(x + 11\pi) - \tan\left(\frac{15\pi}{2} - x\right) - \frac{1}{\cos x \cdot \sin x}$$

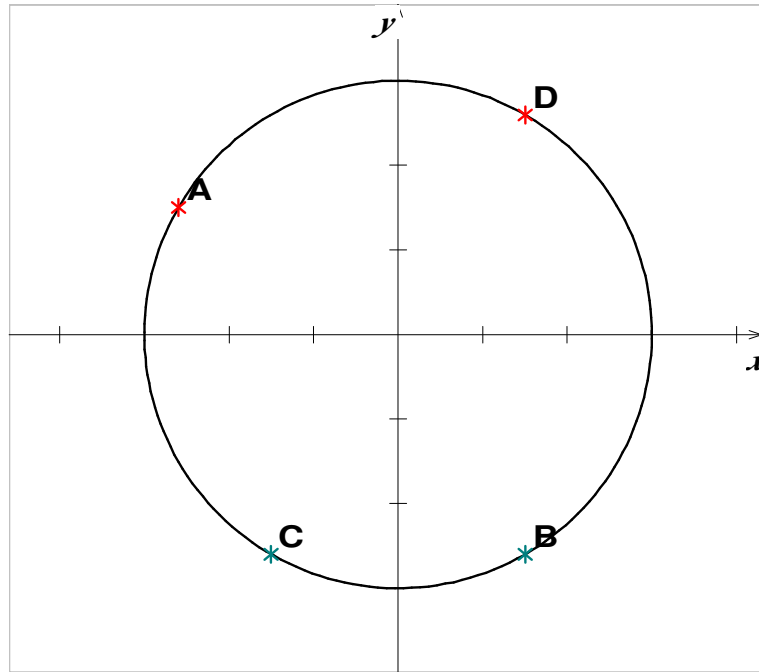
Exercice 4

- 1) On suppose que $x \in \left[\frac{3\pi}{2}, 2\pi\right]$ tel que $\cos x = \frac{2}{5}$. Calculer $\sin x$ et $\tan x$
- 2) On suppose que $x \in \left[\pi, \frac{3\pi}{2}\right]$ tel que $\tan x = \frac{1}{4}$. Calculer $\cos x$ et $\sin x$

3) On suppose que $x \in \left[-\frac{\pi}{2}, 0 \right]$ tel que $\sin x = \frac{-1}{5}$. Calculer $\cos x$ et $\tan x$

Corrigé de l'exercice 1 :

1.



2.

a)

$$\triangleright \cos\left(\frac{5\pi}{6}\right) = \cos\left(\pi - \frac{\pi}{6}\right) = -\cos\left(\frac{\pi}{6}\right) = -\frac{\sqrt{3}}{2}$$

$$\triangleright \cos\left(\frac{-\pi}{3}\right) = \cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$$

$$\triangleright \cos\left(\frac{4\pi}{3}\right) = \cos\left(\pi + \frac{\pi}{3}\right) = -\cos\left(\frac{\pi}{3}\right) = -\frac{1}{2}$$

$$\triangleright \cos\left(\frac{55\pi}{3}\right) = \cos\left(\frac{\pi}{3} + 18\pi\right) = \cos\left(\frac{\pi}{3} + 2(9)\pi\right) = \cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$$

b)

$$\triangleright \sin\left(\frac{5\pi}{6}\right) = \sin\left(\pi - \frac{\pi}{6}\right) = \sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$$

$$\begin{aligned} \triangleright \sin\left(\frac{-\pi}{3}\right) &= -\sin\left(\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2} \\ \triangleright \sin\left(\frac{4\pi}{3}\right) &= \sin\left(\pi + \frac{\pi}{3}\right) = -\sin\left(\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2} \\ \triangleright \sin\left(\frac{55\pi}{3}\right) &= \sin\left(\frac{\pi}{3} + 2(9)\pi\right) = \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2} \end{aligned}$$

c)

$$\begin{aligned} \triangleright \tan\left(\frac{5\pi}{6}\right) &= \tan\left(\pi - \frac{\pi}{6}\right) = -\tan\left(\frac{\pi}{6}\right) = -\frac{\sqrt{3}}{3} \\ \triangleright \tan\left(\frac{-\pi}{3}\right) &= -\tan\left(\frac{\pi}{3}\right) = -\sqrt{3} \\ \triangleright \tan\left(\frac{4\pi}{3}\right) &= \tan\left(\pi + \frac{\pi}{3}\right) = \tan\left(\frac{\pi}{3}\right) = \sqrt{3} \\ \triangleright \tan\left(\frac{55\pi}{3}\right) &= \tan\left(\frac{\pi}{3} + 18\pi\right) = \tan\left(\frac{\pi}{3}\right) = \sqrt{3} \end{aligned}$$

Corrigé de l'exercice 2 :

\triangleright On a :

$$\begin{aligned} \cos^2\left(\frac{9\pi}{22}\right) &= \cos^2\left(\frac{11\pi}{22} - \frac{2\pi}{22}\right) = \cos^2\left(\frac{\pi}{2} - \frac{\pi}{11}\right) = \sin^2\left(\frac{\pi}{11}\right) \\ \cos^2\left(\frac{5\pi}{22}\right) &= \cos^2\left(\frac{11\pi}{22} - \frac{6\pi}{22}\right) = \cos^2\left(\frac{\pi}{2} - \frac{3\pi}{11}\right) = \sin^2\left(\frac{3\pi}{11}\right) \end{aligned}$$

Donc :

$$A = \cos^2\left(\frac{\pi}{11}\right) + \cos^2\left(\frac{3\pi}{11}\right) + \cos^2\left(\frac{5\pi}{22}\right) + \cos^2\left(\frac{9\pi}{22}\right) = \cos^2\left(\frac{\pi}{11}\right) + \cos^2\left(\frac{3\pi}{11}\right) + \sin^2\left(\frac{3\pi}{11}\right) + \sin^2\left(\frac{\pi}{11}\right) = 2$$

\triangleright On a :

$$\begin{aligned} \sin\left(\frac{14\pi}{13}\right) &= \sin\left(\pi + \frac{\pi}{13}\right) = -\sin\left(\frac{\pi}{13}\right) \\ \sin\left(\frac{16\pi}{13}\right) &= \sin\left(\pi + \frac{3\pi}{13}\right) = -\sin\left(\frac{3\pi}{13}\right) \end{aligned}$$

Donc :

$$B = \sin\left(\frac{\pi}{13}\right) + \sin\left(\frac{3\pi}{13}\right) + \sin\left(\frac{14\pi}{13}\right) + \sin\left(\frac{16\pi}{13}\right) = \sin\left(\frac{\pi}{13}\right) + \sin\left(\frac{3\pi}{13}\right) - \sin\left(\frac{\pi}{13}\right) - \sin\left(\frac{3\pi}{13}\right) = 0$$

▷ On a :

$$\tan\left(\frac{4\pi}{7}\right) = \tan\left(\pi - \frac{3\pi}{7}\right) = -\tan\left(\frac{3\pi}{7}\right)$$

$$\tan\left(\frac{6\pi}{7}\right) = \tan\left(\pi - \frac{\pi}{7}\right) = -\tan\left(\frac{\pi}{7}\right)$$

Donc :

$$C = \tan\left(\frac{\pi}{7}\right) + \tan\left(\frac{3\pi}{7}\right) + \tan\left(\frac{4\pi}{7}\right) + \tan\left(\frac{6\pi}{7}\right) = \tan\left(\frac{\pi}{7}\right) + \tan\left(\frac{3\pi}{7}\right) - \tan\left(\frac{3\pi}{7}\right) - \tan\left(\frac{\pi}{7}\right) = 0$$

Corrigé de l'exercice 3 :

$$\triangleright A = \sin(x + 5\pi) + \sin\left(\frac{9\pi}{2} - x\right) + \sin(13\pi - x) + \sin\left(\frac{17\pi}{2} + x\right)$$

On a :

$$\circ \sin(x + 5\pi) = \sin(x + \pi + 2(2)\pi) = \sin(x + \pi) = -\sin x$$

$$\circ \sin\left(\frac{9\pi}{2} - x\right) = \sin\left(\frac{\pi}{2} - x + 2(2)\pi\right) = \sin\left(\frac{\pi}{2} - x\right) = \cos(x)$$

$$\circ \sin(13\pi - x) = \sin(\pi - x + 2(6)\pi) = \sin(\pi - x) = \sin(x)$$

$$\circ \sin\left(\frac{17\pi}{2} + x\right) = \sin\left(\frac{\pi}{2} + x + 2(4)\pi\right) = \sin\left(\frac{\pi}{2} + x\right) = \cos(x)$$

$$\text{Donc : } A = 2\cos(x)$$

$$\triangleright B = \cos(x + 9\pi) + \cos(13\pi - x) + \cos(x + 28\pi) + \cos(x)$$

On a :

$$\circ \cos(x + 9\pi) = \cos(x + \pi + 2(4)\pi) = \cos(x + \pi) = -\cos(x)$$

$$\circ \cos(13\pi - x) = \cos(\pi - x + 2(6)\pi) = \cos(\pi - x) = -\cos(x)$$

$$\circ \cos(x + 28\pi) = \cos(x + 2(14)\pi) = \cos(x)$$

$$\text{Donc : } B = 0$$

$$\triangleright C = \tan(x + 11\pi) - \tan\left(\frac{15\pi}{2} - x\right) - \frac{1}{\cos x \cdot \sin x}$$

On a :

$$\circ \tan(x + 11\pi) = \tan(x)$$

$$\begin{aligned} \circ \quad \tan\left(\frac{15\pi}{2} - x\right) &= \tan\left(\frac{\pi}{2} - x + 7\pi\right) = \tan\left(\frac{\pi}{2} - x\right) = \frac{1}{\tan x} \\ \circ \quad \frac{1}{\cos x \cdot \sin x} &= \frac{1}{\cos^2(x) \times \frac{\sin x}{\cos(x)}} = \frac{1 + \tan^2(x)}{\tan x} = \frac{1}{\tan x} + \tan x \end{aligned}$$

$$\text{Donc } C = \frac{-2}{\tan x}$$

Corrigé de l'exercice 4 :

1) Soit $x \in \left[\frac{3\pi}{2}, 2\pi\right]$, on a :

$$\cos^2(x) + \sin^2(x) = 1$$

$$\text{Donc } \sin^2(x) = 1 - \cos^2(x) = 1 - \left(\frac{2}{5}\right)^2 = \frac{21}{25}$$

$$\text{Donc } \sin x = -\frac{\sqrt{21}}{5} \quad \text{ou} \quad \sin x = \frac{\sqrt{21}}{5}$$

Puisque $x \in \left[\frac{3\pi}{2}, 2\pi\right]$ alors $\sin x \leq 0$

$$\text{Et par suite } \sin x = -\frac{\sqrt{21}}{5}$$

$$\text{De plus : } \tan x = \frac{\sin x}{\cos x} = \frac{-\frac{\sqrt{21}}{5}}{\frac{2}{5}} = -\frac{\sqrt{21}}{2}$$

2) Soit $x \in \left[\pi, \frac{3\pi}{2}\right]$, on a :

$$\cos^2(x) = \frac{1}{1 + \tan^2(x)} = \frac{1}{1 + \left(\frac{1}{4}\right)^2} = \frac{16}{17}$$

$$\text{Donc : } \cos x = -\frac{4\sqrt{17}}{17} \quad \text{ou} \quad \cos x = \frac{4\sqrt{17}}{17}$$

Puisque $x \in \left[\pi, \frac{3\pi}{2}\right]$ alors $\cos(x) \leq 0$

$$\text{Et par suite } \cos(x) = -\frac{4\sqrt{17}}{17}$$

De plus, on sait que $\sin(x) = \cos(x) \times \tan(x)$

$$\text{Donc } \sin(x) = -\frac{4\sqrt{17}}{17} \times \frac{1}{4} = -\frac{\sqrt{17}}{17}$$

3) Soit $x \in \left[-\frac{\pi}{2}, 0\right]$

On sait que $\cos^2(x) + \sin^2(x) = 1$

$$\text{Donc } \cos^2(x) = 1 - \sin^2(x) = 1 - \left(\frac{-1}{5}\right)^2 = \frac{24}{25}$$

$$\text{Donc } \cos x = -\frac{2\sqrt{6}}{5} \quad \text{ou} \quad \cos x = \frac{2\sqrt{6}}{5}$$

Puisque $x \in \left[-\frac{\pi}{2}, 0\right]$ alors $\cos(x) \geq 0$

$$\text{Et par suite } \cos(x) = \frac{2\sqrt{6}}{5}$$

De plus, on sait que : $\tan(x) = \frac{\sin x}{\cos x}$

$$\text{Donc } \tan(x) = \frac{\frac{-1}{5}}{\frac{2\sqrt{6}}{5}} = -\frac{1}{2\sqrt{6}} = -\frac{\sqrt{6}}{12}$$

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