الامتحان الوطني الموحد للبكالوريا
الدورة الاستدراكية 2024
- الموضوع -
SSSSSSSSSSSSSSSSSSSS-SSSS
RS 24E

المملكة المغربية
وزارة التربية الوطنية والتعليم الأولي والرياضة
المركز الوطني للتقويم والامتحانات

4h	مدة الإنجاز	الرياضيات	المادة
9	المعامل	شعبة العلوم الرياضية (أ) و (ب) (خيار انجليزية)	الشعبة المسلك

- The duration of the exam is four hours

-The exam consists of 5 independent exercises

- The exercises can be treated in the order chosen by the candidate

- Exercise1 concerns analysis ……….…………..…(6.5 pts)

- Exercise2 concerns analysis ……….…………..…(3.5 pts)

- Exercise3 concerns complex numbers.............…(3.5 pts)

- Exercise4 concerns algebra ……………...........(3.5 pts)

- Exercise5 concerns arithmetic ………………........(3 pts)

The use of the calculator is not allowed

The use of red color on sheet paper is not permitted

EXERCISE1 : (6.5 points)

Let n be a natural number greater than or equal to 2

Consider the numerical function f_n defined on $[0,+\infty[$ by:

$$f_n(0)=0 \quad \text{and} \quad \forall x\in]0,+\infty[\ ; \ f_n(x)=x-x^n\ln x$$

And let (C_n) be its curve in an orthonormal coordinate system.

0.25 1-a) Show that f_n is continuous on the right at 0

0.75 b) Show that: $\lim\limits_{x\to+\infty} f_n(x)=-\infty$ and $\lim\limits_{x\to+\infty}\frac{f_n(x)}{x}=-\infty$, then interpret graphically the obtained result.

0.5 c) Show that f_n is differentiable on the right at the point 0 and that its derivative on the right at the point 0 is equal to 1

0.5 d) Show that f_n differentiable on $]0,+\infty[$ and that:

$$\forall x\in]0,+\infty[\ ; \ f_n'(x)=1-x^{n-1}-nx^{n-1}\ln x$$

0.5 e) Show that f_n is strictly increasing on $[0;1]$ and strictly decreasing on $[1,+\infty[$

0.5 2-a) Show that for $n\geq 2$, we have: $\forall x\in[0,+\infty[\ ; \ f_{n+1}(x)\leq f_n(x)$

0.25 b) Deduce the relative position of (C_n) and (C_{n+1})

0.5 3-a) Show that for $n\geq 2$, there exists a unique real number $\alpha_n\in]1;2[$ such that: $f_n(\alpha_n)=0$ (We take $\ln 2=0,7$)

0.25 b) Verify that $(\forall n\geq 2)\quad \alpha_{n+1}^n\ln\alpha_{n+1}=1$

0.25 c) Deduce that for $n\geq 2$, $f_n(\alpha_{n+1})=\alpha_{n+1}-1$

0.5 d) Show that the sequence $(\alpha_n)_{n\geq 2}$,thus defined, is strictly decreasing.

0.25 e) Deduce that the sequence $(\alpha_n)_{n\geq 2}$ is convergent.

4- We put: $\ell=\lim\limits_{n\to+\infty}\alpha_n$

0.25 a) Show that: $1\leq \ell\leq 2$

0.5 b) Show that for all $n\geq 2$, $n-1=-\frac{\ln(\ln(\alpha_n))}{\ln(\alpha_n)}$

0.25 c) Suppose that $\ell>1$. Calculate $\lim\limits_{n\to+\infty}\frac{\ln(\ln(\alpha_n))}{\ln(\alpha_n)}$ in terms of ℓ

0.5 d) Deduce the value of the limit ℓ

EXERCISE2 : (3.5 points)

0.25 1-a) Calculate the integral: $\int_0^1 \frac{1}{1+x^2}\,dx$

0.5 b) For all $n\geq 1$, we put: $u_n=\sum_{k=1}^{k=n}\frac{n}{n^2+k^2}$.

Show that the sequence $(u_n)_{n\geq 1}$ is convergent and determine its limit.

0.25 2- Show that: $\int_0^1 \frac{1}{\left(1+x^2\right)^2}\,dx\leq 1$

0.5 3-a) Show that: $\left(\forall x\in[0,1]\right)$; $0\leq e^x-1\leq e.x$

0.25 b) Deduce that: $\left(\forall x\in[0,1]\right)$; $0\leq e^x-1-x\leq \frac{e}{2}x^2$

4- For all $n\geq 1$, we put: $w_n=\sum_{k=1}^{k=n}\left(e^{\frac{n}{n^2+k^2}}-1\right)$

0.25 a) Show that for all $n\geq 1$, we have: $0\leq w_n-u_n\leq \frac{e}{2}\sum_{k=1}^{k=n}\left(\frac{n}{n^2+k^2}\right)^2$

0.25 b) Show that the function: $x\mapsto\left(1+x^2\right)^{-2}$ is strictly decreasing on $[0,1]$

0.25 c) Deduce that for all $n\geq 1$ and for all $k\in\{1,2,......,n\}$, we have:

$$\frac{1}{n}\left(1+\left(\frac{k}{n}\right)^2\right)^{-2}\leq\int_{\frac{k-1}{n}}^{\frac{k}{n}}\left(1+x^2\right)^{-2}dx$$

0.5 5-a) Show that for all $n\geq 1$, we have: $0\leq w_n-u_n\leq\frac{e}{2n}$

0.5 b) Deduce that the sequence $\left(w_n\right)_{n\geq 1}$ is convergent and determine its limit.

EXERCISE3 : (3.5 points)

Let $m\in\mathbb{C}^*$

Part I: Consider in $\mathbb{C}$ the equation with variable z

$$(E)\ :\ z^2-(2+i)mz+m^2(1+i)=0$$

0.25 1- a) Verify that the discriminant of the equation (E) is $\Delta=(im)^2$

0.5 b) Solve in $\mathbb{C}$ the equation (E)

2- Let z_1 and z_2 be the solutions of (E)

0.5 Write z_1z_2 in the exponential form in case where $m=re^{i\theta}$ ($r\in\mathbb{R}_+^*$, $\theta\in\mathbb{R}$)

Part II: The complex plane is attached to the direct orthonormal coordinate system $(O,\vec{e_1},\vec{e_2})$

We put $z_1 = m$ and $z_2 = m(1+i)$

Let M_1 be the point with affix z_1 , M_2 the point with affix z_2 and $M_3(z_3)$ the image of O by the rotation with center M_2 and angle $\left(-\frac{\pi}{2}\right)$ and $M_4(z_4)$ the image of the point M_1 by the homothety with center O and ratio k $\left(k \in \mathbb{R}^* - \{1\}\right)$

0.75 1- Calculate z_3 in terms of m and calculate z_4 in terms of m and k

0.75 2- Give the algebraic form of $\frac{z_4 - z_2}{z_4 - z_1} \times \frac{z_3 - z_1}{z_3 - z_2}$

0.75 3- Deduce that the points M_1, M_2, M_3 and M_4 are cocyclic if and only if $k = -2$

<u>EXERCISE4</u> : (3.5 points)

We provide the set $\mathbb{C}$ of complex numbers with the internal law $*$ defined by:

$$\forall (x,x',y,y') \in \mathbb{R}^4 \ ; \quad (x+iy)*(x'+iy') = (xy' + y^5 x') + iyy'$$

Part I:

0.25 1- a) Verify that: $1 * 2i = 2$

0.25 b) Show that the internal law $*$ is not commutative.

0.5 2- Show that the internal law $*$ is associative.

0.25 3- a) Verify that: $1*(1+2i) = 2$

0.25 b) Deduce that $(\mathbb{C}, *)$ is not a group.

4- Let E be the sub-set of $\mathbb{C}$ defined by $E = \left\{x + yi \,/\, x \in \mathbb{R} \text{ et } y \in \mathbb{R}^*\right\}$

0.25 a) Show that E is stable in $(\mathbb{C}, *)$

0.5 b) Show that $(E, *)$ is a non-commutative group.

Part II:

Consider the sub-sets of E defined by: $F = \left\{yi \,/\, y \in \mathbb{R}^*\right\}$ and $G = \left\{x + i \,/\, x \in \mathbb{R}\right\}$

0.5 1- Show that F is a sub-group of $(E, *)$

2- Consider the application φ defined from $\mathbb{R}$ to $\mathbb{C}$ by : $(\forall x \in \mathbb{R})$; $\varphi(x) = x + i$

0.25 a) Show that: $\varphi(\mathbb{R}) = G$

0.25 b) Show that φ is a homomorphism from $(\mathbb{R},+)$ to $(\mathbb{C}, *)$

0.25 c) Deduce that $(G,*)$ is a commutative group.

EXERCISE5 :(3 points)

0.5 1- Using Euclid's algorithm, determine the integer $u \in \{1,2,...,22\}$ such that: $10u \equiv 1 \ [23]$

2- Let m be a natural number and q and r, respectively, the quotient and the remainder of the Euclidean division of m by 10

0.5 a) Show that: $m \equiv 10(q+ur) \ [23]$

0.75 b) Show that: 23 divides $m \Leftrightarrow$ 23 divides $(q+ur)$

3- we consider in $\mathbb{N}$ the system $(S):\begin{cases} x \equiv 1 \ [23] \\ x \equiv 2 \ [10] \end{cases}$

0.75 a) Show that if x is a solution of the system (S) then there exists $q \in \mathbb{N}$ such that $x = 10q+2$ and 23 divides $(q+7)$

0.5 b) Solve in $\mathbb{N}$ the system (S)

END