

تمرين 1 :

$$\sin(2x) + \sin(-5x) = 0$$

$$\sin 3x - \cos x = 0$$

$$\sqrt{3} \tan x + 1 = 0$$

$$\sin 2x - 2 \cos^2 x = 0$$

حل في IR المعادلات التالية :**تمرين 2 :**

$$\sin(x) + \sqrt{3} \cos x = \sqrt{3}$$

حل في IR المعادلات التالية :

$$\frac{\cos x}{\sqrt{3}} - \sin x = 2$$

تمرين 3 :

$$2 \sin^2 x + 3 \cos x = 3$$

$$\frac{\sin x}{2 + \cos x} = \frac{\cos x}{2 + \sin x}$$

$$\tan x = \sin 2x$$

$$\cos 2x - 7 \sin x = 4$$

$$\sin 5x - \sin 3x = \cos 6x + \cos 2x$$

$$2 \sin^2 x + \sqrt{3} \sin 2x = 3$$

$$\sqrt{2} \sin\left(x - \frac{f}{3}\right) - \sin x = \cos x$$

حل في IR المعادلات التالية :**تمرين 4 :**

$$\forall (a, b) \in IR^2 \quad \cos(a+b) \sin(a-b) = \sin a \cos a - \sin b \cos b \quad (1) \text{ بين أن :}$$

$$\cos\left(x + \frac{f}{4}\right) \times \sin\left(x - \frac{f}{4}\right) = -\frac{1}{4} \quad (2) \text{ حل في } IR \text{ المعادلة :}$$

$$(E): \sqrt{3} \sin(x) + \cos x = 1 \quad \text{نعتبر المعادلة :}$$

1) حل في IR المعادلة (E) .2) تحقق أن العدد : $x_k = f + 2kf \quad / k \in Z$ ليس حلا للمعادلة (E) .

$$3) \text{ نضع } t = \tan \frac{x}{2} \text{ ، بين أن : } t^2 - \sqrt{3}t = 0$$

4) استنتج من جديد حلول المعادلة (E)

تمرين 1 :

$$\sin(2x) + \sin(-5x) = 0 \Leftrightarrow \sin(2x) = \sin(5x)$$

$$\sin(2x) + \sin(-5x) = 0 \Leftrightarrow 2x = 5x + 2kf / k \in \mathbb{Z} \text{ ou } 2x = f - 5x + 2kf / k \in \mathbb{Z} \text{ لدينا}$$

$$\sin(2x) + \sin(-5x) = 0 \Leftrightarrow -3x = 2kf / k \in \mathbb{Z} \text{ ou } 7x = (2k+1)f / k \in \mathbb{Z}$$

$$S = \left\{ \frac{2k}{3}f / k \in \mathbb{Z} \right\} \cup \left\{ \frac{(2k+1)f}{7} / k \in \mathbb{Z} \right\} \text{ : أو أيضا } S = \left\{ -\frac{2k}{3}f / k \in \mathbb{Z} \right\} \cup \left\{ \frac{(2k+1)f}{7} / k \in \mathbb{Z} \right\} \text{ : بالتالي}$$

يمكن أحيانا تبسيط تعبير مجموعة الحلول (أعلاه : $-k \in \mathbb{Z} \Leftrightarrow k \in \mathbb{Z}$)

$$\sin 3x - \cos x = 0 \Leftrightarrow \sin 3x = \cos x \Leftrightarrow \cos\left(\frac{f}{2} - 3x\right) = \cos(x)$$

$$\sin 3x - \cos x = 0 \Leftrightarrow \frac{f}{2} - 3x = x + 2kf / k \in \mathbb{Z} \text{ ou } \frac{f}{2} - 3x = -x + 2kf / k \in \mathbb{Z}$$

$$\sin 3x - \cos x = 0 \Leftrightarrow 4x = \frac{f}{2} - 2kf / k \in \mathbb{Z} \text{ ou } 2x = \frac{f}{2} - 2kf / k \in \mathbb{Z}$$

$$\sin 3x - \cos x = 0 \Leftrightarrow x = \frac{f}{8} - \frac{kf}{2} / k \in \mathbb{Z} \text{ ou } x = \frac{f}{4} - kf / k \in \mathbb{Z}$$

$$S = \left\{ \frac{f}{8} + \frac{kf}{2} / k \in \mathbb{Z} \right\} \cup \left\{ \frac{f}{4} + kf / k \in \mathbb{Z} \right\} \text{ : أو أيضا } S = \left\{ \frac{f}{8} - \frac{kf}{2} / k \in \mathbb{Z} \right\} \cup \left\{ \frac{f}{4} - kf / k \in \mathbb{Z} \right\} \text{ : بالتالي}$$

$$D = \mathbb{R} - \left\{ \frac{f}{2} + kf / k \in \mathbb{Z} \right\} \text{ : مجموعة صلاحية المعادلة هي}$$

$$\sqrt{3} \tan x + 1 = 0 \Leftrightarrow \tan x = \tan\left(\frac{-f}{6}\right) \Leftrightarrow x = \frac{-f}{6} + kf / k \in \mathbb{Z} \text{ لدينا في هذه المجموعة :}$$

$$S = \left\{ \frac{-f}{6} + kf / k \in \mathbb{Z} \right\} \cap D = \left\{ \frac{-f}{6} + kf / k \in \mathbb{Z} \right\} \text{ : بالتالي}$$

$$\sin 2x - 2\cos^2 x = 0 \Leftrightarrow 2\sin x \cos x - 2\cos^2 x = 0$$

$$\sin 2x - 2\cos^2 x = 0 \Leftrightarrow 2\cos x(\sin x - \cos x) = 0$$

$$\sin 2x - 2\cos^2 x = 0 \Leftrightarrow \cos x = 0 \text{ ou } \cos(x) = \sin(x)$$

$$\sin 2x - 2\cos^2 x = 0 \Leftrightarrow \cos x = 0 \text{ ou } \cos(x) = \cos\left(\frac{f}{2} - x\right)$$

$$\sin 2x - 2\cos^2 x = 0 \Leftrightarrow \left(x = \frac{f}{2} + kf / k \in \mathbb{Z}\right) \text{ ou } \left(\frac{f}{2} - x = x + 2kf / k \in \mathbb{Z}\right) \text{ ou } \left(\frac{f}{2} - x = -x + 2kf / k \in \mathbb{Z}\right)$$

$$\sin 2x - 2\cos^2 x = 0 \Leftrightarrow \left(x = \frac{f}{2} + kf / k \in \mathbb{Z}\right) \text{ ou } \left(2x = \frac{f}{2} - 2kf / k \in \mathbb{Z}\right) \text{ ou } \left(\frac{f}{2} = 2kf / k \in \mathbb{Z}\right)$$

$$\sin 2x - 2\cos^2 x = 0 \Leftrightarrow \left(x = \frac{f}{2} + kf / k \in \mathbb{Z}\right) \text{ ou } \left(x = \frac{f}{4} - kf / k \in \mathbb{Z}\right) \text{ ou } \left(k = \frac{1}{4} / k \in \mathbb{Z}\right)$$

$$\sin 2x - 2\cos^2 x = 0 \Leftrightarrow \left(x = \frac{f}{2} + kf / k \in \mathbb{Z}\right) \text{ ou } \left(x = \frac{f}{4} + kf / k \in \mathbb{Z}\right)$$

$$S = \left\{ \frac{f}{2} + kf / k \in \mathbb{Z} \right\} \cup \left\{ \frac{f}{4} + kf / k \in \mathbb{Z} \right\} \text{ : بالتالي}$$

$$\cos a = \cos b \Leftrightarrow a = b + 2kf \text{ ou } a = -b + 2kf \quad / k \in \mathbb{Z}$$

$$\cos a = 1 \Leftrightarrow a = 2kf / k \in \mathbb{Z} \quad ; \quad \cos a = -1 \Leftrightarrow a = f + 2kf / k \in \mathbb{Z} \quad ; \quad \cos a = 0 \Leftrightarrow a = \frac{f}{2} + kf / k \in \mathbb{Z}$$

$$\sin a = \sin b \Leftrightarrow a = b + 2kf / k \in \mathbb{Z} \text{ ou } a = f - b + 2kf / k \in \mathbb{Z}$$

$$\sin a = 1 \Leftrightarrow a = \frac{f}{2} + 2kf / k \in \mathbb{Z} \quad ; \quad \sin a = -1 \Leftrightarrow a = \frac{-f}{2} + 2kf / k \in \mathbb{Z} \quad ; \quad \sin a = 0 \Leftrightarrow a = kf / k \in \mathbb{Z}$$

$$\tan a = \tan b \Leftrightarrow a = b + kf / k \in \mathbb{Z}$$

تمرين 2:

$$\sin(x) + \sqrt{3} \cos x = \sqrt{3} \Leftrightarrow \frac{1}{2} \sin(x) + \frac{\sqrt{3}}{2} \cos x = \frac{\sqrt{3}}{2}$$

$$\sin(x) + \sqrt{3} \cos x = \sqrt{3} \Leftrightarrow \sin\left(\frac{f}{6}\right) \sin(x) + \cos\left(\frac{f}{6}\right) \cos x = \frac{\sqrt{3}}{2}$$

$$\sin(x) + \sqrt{3} \cos x = \sqrt{3} \Leftrightarrow \cos\left(x - \frac{f}{6}\right) = \cos\left(\frac{f}{6}\right)$$

لدينا:

$$\sin(x) + \sqrt{3} \cos x = \sqrt{3} \Leftrightarrow x - \frac{f}{6} = \frac{f}{6} + 2kf / k \in \mathbb{Z} \text{ ou } x - \frac{f}{6} = \frac{-f}{6} + 2kf / k \in \mathbb{Z}$$

$$\sin(x) + \sqrt{3} \cos x = \sqrt{3} \Leftrightarrow x = \frac{f}{3} + 2kf / k \in \mathbb{Z} \text{ ou } x = 2kf / k \in \mathbb{Z}$$

$$S = \left\{ \frac{f}{3} + 2kf / k \in \mathbb{Z} \right\} \cup \{ 2kf / k \in \mathbb{Z} \} : \text{بالتالي}$$

$$\sin x - \cos x = -1 \Leftrightarrow \frac{1}{\sqrt{2}} \sin x - \frac{1}{\sqrt{2}} \cos x = -\frac{1}{\sqrt{2}}$$

$$\sin x - \cos x = -1 \Leftrightarrow \frac{1}{\sqrt{2}} \cos x - \frac{1}{\sqrt{2}} \sin x = \frac{1}{\sqrt{2}}$$

$$\sin x - \cos x = -1 \Leftrightarrow \cos\left(\frac{f}{4}\right) \cos x - \cos\left(\frac{f}{4}\right) \sin x = \cos\left(\frac{f}{4}\right)$$

لدينا:

$$\sin x - \cos x = -1 \Leftrightarrow \cos\left(x + \frac{f}{4}\right) = \cos\left(\frac{f}{4}\right)$$

$$\sin x - \cos x = -1 \Leftrightarrow x + \frac{f}{4} = \frac{f}{4} + 2kf / k \in \mathbb{Z} \text{ ou } x + \frac{f}{4} = \frac{-f}{4} + 2kf / k \in \mathbb{Z}$$

$$\sin x - \cos x = -1 \Leftrightarrow x = 2kf / k \in \mathbb{Z} \text{ ou } x = \frac{-f}{2} + 2kf / k \in \mathbb{Z}$$

$$S = \left\{ \frac{-f}{2} + 2kf / k \in \mathbb{Z} \right\} \cup \{ 2kf / k \in \mathbb{Z} \} : \text{بالتالي}$$

$$\frac{\cos x}{\sqrt{3}} - \sin x = 2 \Leftrightarrow \cos x - \sqrt{3} \sin x = 2\sqrt{3}$$

$$\frac{\cos x}{\sqrt{3}} - \sin x = 2 \Leftrightarrow \frac{1}{2} \cos x - \frac{\sqrt{3}}{2} \sin x = \sqrt{3}$$

بما أن $\sqrt{3} > 1$ فإن: $S = \emptyset$

$$\frac{\cos x}{\sqrt{3}} - \sin x = 2 \Leftrightarrow \cos\left(\frac{f}{3}\right) \cos x - \sin\left(\frac{f}{3}\right) \sin x = \sqrt{3}$$

لدينا:

$$\frac{\cos x}{\sqrt{3}} - \sin x = 2 \Leftrightarrow \cos\left(x + \frac{f}{3}\right) = \sqrt{3}$$

تمرين 3 :

$$2 \sin^2 x + 3 \cos x = 3 \Leftrightarrow 2(1 - \cos^2 x) + 3 \cos x = 3 \Leftrightarrow 2 \cos^2 x - 3 \cos x + 1 = 0 \quad \text{لدينا :}$$

$$t = \frac{3-1}{4} = \frac{1}{2} \quad \text{أو} \quad t = \frac{3+1}{4} = 1 \quad \text{منه :} \quad \Delta = 9 - 8 = 1, \quad 2t^2 - 3t + 1 = 0 \quad \text{منه :} \quad t = \cos x$$

$$S = \left\{ \frac{f}{3} + 2kf / k \in \mathbb{Z} \right\} \cup \left\{ -\frac{f}{3} + 2kf / k \in \mathbb{Z} \right\} \cup \{2kf / k \in \mathbb{Z}\} \quad \text{بالتالي :} \quad \cos = \frac{1}{2} \quad \text{أو} \quad \cos x = 1$$

بما أن $2 + \cos x \geq 1 > 0$ و $2 + \sin x \geq 1 > 0$ فمجموعة صلاحية المعادلة هي : IR

$$\frac{\sin x}{2 + \cos x} = \frac{\cos x}{2 + \sin x} \Leftrightarrow 2 \sin x + \sin^2 x = 2 \cos x + \cos^2 x$$

$$\frac{\sin x}{2 + \cos x} = \frac{\cos x}{2 + \sin x} \Leftrightarrow 2(\sin x - \cos x) + \sin^2 x - \cos^2 x = 0$$

$$\frac{\sin x}{2 + \cos x} = \frac{\cos x}{2 + \sin x} \Leftrightarrow (\sin x - \cos x)(2 + \sin x + \cos x) = 0$$

$$\frac{\sin x}{2 + \cos x} = \frac{\cos x}{2 + \sin x} \Leftrightarrow \cos x = \sin x \quad \text{ou} \quad \sin x + \cos x = -2$$

$$\frac{\sin x}{2 + \cos x} = \frac{\cos x}{2 + \sin x} \Leftrightarrow \cos x = \cos\left(\frac{f}{2} - x\right) \quad \text{ou} \quad \frac{1}{\sqrt{2}} \sin x + \frac{1}{\sqrt{2}} \cos x = \frac{-2}{\sqrt{2}}$$

$$\frac{\sin x}{2 + \cos x} = \frac{\cos x}{2 + \sin x} \Leftrightarrow x = \frac{f}{2} - x + 2kf / k \in \mathbb{Z} \quad \text{ou} \quad x = \frac{-f}{2} + x + 2kf / k \in \mathbb{Z}$$

$$\text{ou} \quad \sin\left(\frac{f}{4}\right) \sin x + \cos\left(\frac{f}{4}\right) \cos x = -\sqrt{2}$$

$$\frac{\sin x}{2 + \cos x} = \frac{\cos x}{2 + \sin x} \Leftrightarrow 2x = \frac{f}{2} + 2kf / k \in \mathbb{Z} \quad \text{ou} \quad \frac{f}{2} = 2kf / k \in \mathbb{Z} \quad \text{ou} \quad \underbrace{\cos\left(x - \frac{f}{4}\right) = -\sqrt{2}}_{\text{impossible, car } -\sqrt{2} < -1}$$

$$\frac{\sin x}{2 + \cos x} = \frac{\cos x}{2 + \sin x} \Leftrightarrow x = \frac{f}{4} + kf / k \in \mathbb{Z} \quad \text{ou} \quad \underbrace{k = \frac{1}{4} / k \in \mathbb{Z}}_{\text{impossible, car } \frac{1}{4} \notin \mathbb{Z}}$$

الآن لدينا :

$$S = \left\{ \frac{f}{4} + kf / k \in \mathbb{Z} \right\} \quad \text{بالتالي :}$$

$$D = IR - \left\{ \frac{f}{2} + kf / k \in \mathbb{Z} \right\} \quad \text{مجموعة صلاحية المعادلة هي :}$$

لدينا في هذه المجموعة :

$$\tan x = \sin 2x \Leftrightarrow \frac{\sin x}{\cos x} = 2 \sin x \cos x \Leftrightarrow \sin x = 2 \sin x \cos^2 x \Leftrightarrow \sin x (1 - 2 \cos^2 x) = 0$$

$$\tan x = \sin 2x \Leftrightarrow \sin x = 0 \quad \text{ou} \quad \cos x = \frac{\sqrt{2}}{2} \quad \text{ou} \quad \cos x = \frac{-\sqrt{2}}{2}$$

بالتالي :

$$S = \left\{ kf / k \in \mathbb{Z} \right\} \cup \left\{ \frac{f}{4} + 2kf / k \in \mathbb{Z} \right\} \cup \left\{ -\frac{f}{4} + 2kf / k \in \mathbb{Z} \right\} \cup \left\{ \frac{3f}{4} + 2kf / k \in \mathbb{Z} \right\} \cup \left\{ -\frac{3f}{4} + 2kf / k \in \mathbb{Z} \right\}$$

$$\tan x = \sin 2x \Leftrightarrow \frac{\sin x}{\cos x} = 2 \sin x \cos x \Leftrightarrow \sin x = 2 \sin x \cos^2 x \Leftrightarrow \sin x (1 - 2 \cos^2 x) = 0$$

$$\tan x = \sin 2x \Leftrightarrow -\sin x \cos(2x) = 0 \Leftrightarrow \sin x = 0 \text{ ou } \cos 2x = 0$$

$$\tan x = \sin 2x \Leftrightarrow x = kf / k \in \mathbb{Z} \text{ ou } 2x = \frac{f}{2} + kf / k \in \mathbb{Z}$$

أو أيضا :

$$\tan x = \sin 2x \Leftrightarrow x = kf / k \in \mathbb{Z} \text{ ou } x = \frac{f}{4} + \frac{kf}{2} / k \in \mathbb{Z}$$

$$S = \{kf / k \in \mathbb{Z}\} \cup \left\{ \frac{f}{4} + \frac{kf}{2} / k \in \mathbb{Z} \right\} : \text{بالتالي}$$

رغم أنه يبدو اختلاف حلي الطريقتين إلا أنهما في الحقيقة يمثلان نفس المجموعة

الطريقة الثانية أفضل لكنها تتطلب ملاحظة بعض الصيغ المثلية الهامة: $\cos 2x = 2 \cos^2 x - 1$

$$\cos 2x - 7 \sin x = 4 \Leftrightarrow 1 - 2 \sin^2 x - 7 \sin x - 4 = 0 \Leftrightarrow 2 \sin^2 x + 7 \sin x + 3 = 0$$

$$\text{نضع } t = \sin x : \text{ فنجد: } 2t^2 + 7t + 3 = 0, \Delta = 49 - 24 = 25 \text{ , منه: } t = \frac{-7-5}{4} = -3 \text{ أو } t = \frac{-7+5}{4} = \frac{-1}{2}$$

$$\text{منه: } \cos x = -3 \text{ (غير ممكن لأن } -3 < -1 \text{)} \text{ أو } \cos x = \frac{-1}{2}$$

$$S = \left\{ \frac{2f}{3} + 2kf / k \in \mathbb{Z} \right\} \cup \left\{ -\frac{2f}{3} + 2kf / k \in \mathbb{Z} \right\} : \text{بالتالي}$$

لدينا :

$$\sin 5x - \sin 3x = \cos 6x + \cos 2x \Leftrightarrow 2 \cos(4x) \sin(x) = 2 \cos(4x) \cos(2x)$$

$$\sin 5x - \sin 3x = \cos 6x + \cos 2x \Leftrightarrow \cos(4x) (\sin(x) - \cos(2x)) = 0$$

$$\sin 5x - \sin 3x = \cos 6x + \cos 2x \Leftrightarrow (\cos 4x = 0) \text{ ou } (\cos 2x = \sin x)$$

$$\sin 5x - \sin 3x = \cos 6x + \cos 2x \Leftrightarrow (4x = 2kf / k \in \mathbb{Z}) \text{ ou } \left(\cos 2x = \cos \left(\frac{f}{2} - x \right) \right)$$

$$\sin 5x - \sin 3x = \cos 6x + \cos 2x \Leftrightarrow \left(x = \frac{kf}{2} / k \in \mathbb{Z} \right) \text{ ou } \left(2x = \frac{f}{2} - x + 2kf / k \in \mathbb{Z} \right) \text{ ou } \left(2x = \frac{-f}{2} + x + 2kf / k \in \mathbb{Z} \right)$$

$$\sin 5x - \sin 3x = \cos 6x + \cos 2x \Leftrightarrow \left(x = \frac{kf}{2} / k \in \mathbb{Z} \right) \text{ ou } \left(x = \frac{f}{6} + \frac{2kf}{3} / k \in \mathbb{Z} \right) \text{ ou } \left(x = \frac{-f}{2} + 2kf / k \in \mathbb{Z} \right)$$

$$S = \left\{ \frac{2f}{3} + 2kf / k \in \mathbb{Z} \right\} \cup \left\{ -\frac{2f}{3} + 2kf / k \in \mathbb{Z} \right\} : \text{بالتالي}$$

$$2 \sin^2 x + \sqrt{3} \sin 2x = 3 \Leftrightarrow 1 - \cos 2x + \sqrt{3} \sin 2x = 3$$

$$2 \sin^2 x + \sqrt{3} \sin 2x = 3 \Leftrightarrow \cos 2x - \sqrt{3} \sin 2x = -2$$

$$2 \sin^2 x + \sqrt{3} \sin 2x = 3 \Leftrightarrow \frac{1}{2} \cos 2x - \frac{\sqrt{3}}{2} \sin 2x = -1$$

$$S = \left\{ \frac{f}{3} + kf / k \in \mathbb{Z} \right\} : \text{بالتالي} \quad 2 \sin^2 x + \sqrt{3} \sin 2x = 3 \Leftrightarrow \cos\left(\frac{f}{3}\right) \cos 2x - \sin\left(\frac{f}{3}\right) \sin 2x = -1 \quad \text{لدينا :}$$

$$2 \sin^2 x + \sqrt{3} \sin 2x = 3 \Leftrightarrow \cos\left(2x + \frac{f}{3}\right) = -1$$

$$2 \sin^2 x + \sqrt{3} \sin 2x = 3 \Leftrightarrow 2x + \frac{f}{3} = f + 2kf / k \in \mathbb{Z}$$

$$2 \sin^2 x + \sqrt{3} \sin 2x = 3 \Leftrightarrow x = \frac{f}{3} + kf / k \in \mathbb{Z}$$

لدينا :

$$\sqrt{2} \sin\left(x - \frac{f}{3}\right) - \sin x = \cos x \Leftrightarrow \sqrt{2} \sin\left(x - \frac{f}{3}\right) = \cos x + \sin x$$

$$\sqrt{2} \sin\left(x - \frac{f}{3}\right) - \sin x = \cos x \Leftrightarrow \sin\left(x - \frac{f}{3}\right) = \frac{1}{\sqrt{2}} \cos x + \frac{1}{\sqrt{2}} \sin x$$

$$\sqrt{2} \sin\left(x - \frac{f}{3}\right) - \sin x = \cos x \Leftrightarrow \sin\left(x - \frac{f}{3}\right) = \sin\left(\frac{f}{4}\right) \cos x + \cos\left(\frac{f}{4}\right) \sin x$$

$$\sqrt{2} \sin\left(x - \frac{f}{3}\right) - \sin x = \cos x \Leftrightarrow \sin\left(x - \frac{f}{3}\right) = \sin\left(x + \frac{f}{4}\right)$$

$$\sqrt{2} \sin\left(x - \frac{f}{3}\right) - \sin x = \cos x \Leftrightarrow x - \frac{f}{3} = x + \frac{f}{4} + 2kf / k \in \mathbb{Z} \text{ ou } x - \frac{f}{3} = f - \left(x + \frac{f}{4}\right) + 2kf / k \in \mathbb{Z}$$

$$\sqrt{2} \sin\left(x - \frac{f}{3}\right) - \sin x = \cos x \Leftrightarrow -\frac{7f}{12} = 2kf / k \in \mathbb{Z} \text{ ou } 2x = \frac{13f}{12} + 2kf / k \in \mathbb{Z}$$

$$\sqrt{2} \sin\left(x - \frac{f}{3}\right) - \sin x = \cos x \Leftrightarrow k = \frac{7}{24} / k \in \mathbb{Z} \text{ ou } x = \frac{13f}{24} + kf / k \in \mathbb{Z}$$

$$S = \left\{ \frac{13f}{24} + kf / k \in \mathbb{Z} \right\} : \text{بالتالي}$$

تمرين 4 :

لدينا لكل $(a, b) \in \mathbb{R}^2$ ، نضع : $A = \cos(a+b) \sin(a-b)$

$$A = (\cos a \cos b - \sin a \sin b)(\sin a \cos b - \cos a \sin b)$$

$$= \cos a \cos b \sin a \cos b - \cos a \cos b \cos a \sin b - \sin a \sin b \sin a \cos b + \sin a \sin b \cos a \sin b$$

$$= \sin a \cos a \cos^2 b - \sin b \cos b \cos^2 a - \sin b \cos b \sin^2 a + \sin a \cos a \sin^2 b$$

$$= \sin a \cos a (\cos^2 b + \sin^2 b) - \sin b \cos b (\cos^2 a + \sin^2 a)$$

$$A = \sin a \cos a - \sin b \cos b$$

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لدينا حسب السؤال السابق :

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$$\cos\left(x + \frac{f}{4}\right) \times \sin\left(x - \frac{f}{4}\right) = -\frac{1}{4} \Leftrightarrow \sin x \cos x - \sin\left(\frac{f}{4}\right) \cos\left(\frac{f}{4}\right) = -\frac{1}{4}$$

$$\cos\left(x + \frac{f}{4}\right) \times \sin\left(x - \frac{f}{4}\right) = -\frac{1}{4} \Leftrightarrow \frac{1}{2} \sin 2x - \frac{1}{2} = -\frac{1}{4}$$

$$\cos\left(x + \frac{f}{4}\right) \times \sin\left(x - \frac{f}{4}\right) = -\frac{1}{4} \Leftrightarrow \frac{1}{2} \sin 2x = \frac{1}{4}$$

$$\cos\left(x + \frac{f}{4}\right) \times \sin\left(x - \frac{f}{4}\right) = -\frac{1}{4} \Leftrightarrow \sin 2x = \frac{1}{2}$$

$$\cos\left(x + \frac{f}{4}\right) \times \sin\left(x - \frac{f}{4}\right) = -\frac{1}{4} \Leftrightarrow 2x = \frac{f}{6} + 2kf / k \in \mathbb{Z} \text{ ou } 2x = \frac{5f}{6} + 2kf / k \in \mathbb{Z}$$

$$\cos\left(x + \frac{f}{4}\right) \times \sin\left(x - \frac{f}{4}\right) = -\frac{1}{4} \Leftrightarrow x = \frac{f}{12} + kf / k \in \mathbb{Z} \text{ ou } x = \frac{5f}{12} + kf / k \in \mathbb{Z}$$

$$S = \left\{ \frac{f}{12} + kf / k \in \mathbb{Z} \right\} \cup \left\{ \frac{5f}{12} + kf / k \in \mathbb{Z} \right\} \text{ بالتالي}$$

تمرين 5: نعتبر المعادلة: $(E): \sqrt{3} \sin(x) + \cos x = 1$

$$(E) \Leftrightarrow \frac{\sqrt{3}}{2} \sin(x) + \frac{1}{2} \cos x = \frac{1}{2}$$

$$(E) \Leftrightarrow \sin\left(\frac{f}{3}\right) \sin(x) + \cos\left(\frac{f}{3}\right) \cos x = \cos\left(\frac{f}{3}\right)$$

$$(E) \Leftrightarrow \cos\left(x - \frac{f}{3}\right) = \cos\left(\frac{f}{3}\right) \text{ لدينا}$$

$$(E) \Leftrightarrow x - \frac{f}{3} = \frac{f}{3} + 2kf / k \in \mathbb{Z} \text{ ou } x - \frac{f}{3} = -\frac{f}{3} + 2kf / k \in \mathbb{Z}$$

$$(E) \Leftrightarrow x = \frac{2f}{3} + 2kf / k \in \mathbb{Z} \text{ ou } x = 2kf / k \in \mathbb{Z}$$

$$S = \left\{ \frac{2f}{3} + 2kf / k \in \mathbb{Z} \right\} \cup \{2kf / k \in \mathbb{Z}\} \text{ بالتالي}$$

لدينا:

$$\frac{\sqrt{3}}{2} \sin(x_k) + \frac{1}{2} \cos(x_k) = \frac{\sqrt{3}}{2} \sin(f + 2kf) + \frac{1}{2} \cos(f + 2kf) = \frac{\sqrt{3}}{2} \sin(f) + \frac{1}{2} \cos(f) = \frac{-1}{2} \text{ 2}$$

إذن $x_k = f + 2kf$ ليس حلا للمعادلة (E) .

بما أن أي حل لمعادلة يحقق: $x \neq f + 2kf$ فإن $\frac{x}{2} \neq \frac{f}{2} + kf$ لكل $k \in \mathbb{Z}$

إذن يمكننا أن نضع: $t = \tan \frac{x}{2}$ ، لدينا الآن: $\sin x = \frac{2t}{1+t^2}$ و $\cos x = \frac{1-t^2}{1+t^2}$

$$(E) \Leftrightarrow \frac{\sqrt{3}}{2} \frac{2t}{1+t^2} + \frac{1-t^2}{1+t^2} = \frac{1}{2} \text{ 3}$$

$$(E) \Leftrightarrow 2\sqrt{3}t + 1 - t^2 = 1 + t^2 \text{ منه:}$$

$$(E) \Leftrightarrow 2t^2 - 2\sqrt{3}t = 0$$

$$(E) \Leftrightarrow t^2 - \sqrt{3}t = 0$$

الآن لدينا : $t=0$ أو $t=\sqrt{3}$ منه : $\tan\left(\frac{x}{2}\right)=0$ أو $\tan\left(\frac{x}{2}\right)=\sqrt{3}$

منه : $\frac{x}{2}=kf/k \in Z$ أو $\frac{x}{2}=\frac{f}{3}+kf/k \in Z$ منه : $x=2kf/k \in Z$ أو $x=\frac{2f}{3}+2kf/k \in Z$

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بالتالي : $S = \left\{ \frac{2f}{3} + 2kf / k \in Z \right\} \cup \{2kf / k \in Z\}$