السنة 1 بكالوريا علوم تجريبية	الحساب المثلثي	سلسلة 3

تمرين1 : حل في IR المعادلات التالية:

$\sqrt{3}\cos x - \sin x = \sqrt{2}$	$\cos x - \sqrt{3}\sin x = 1$	$\cos x - \sin x = \sqrt{2}$
$\sin x + \cos x = 1$	$\sin x + \cos x = \sqrt{2}$	$\cos\dfrac{x}{2} - \sin\dfrac{x}{2} = -1$

تمرين2 : نعتبر الدالة : $f(x) = \dfrac{\cos x + \sin x}{\cos x - \sin x}$

1) حل في IR المعادلة : $\cos x - \sin x = 0$

2) حدد Df

3) بين أن : $\forall x \in Df \quad f(x) = \dfrac{1 + \sin 2x}{\cos 2x}$

4) حل في IR المعادلة : $f(x) - \sqrt{3} = 0$

تمرين3 : ليكن x عددا حقيقيا، نعتبر التعبير : $A(x) = \sqrt{3}\cos\left(2x - \dfrac{\pi}{3}\right) - \sin\left(2x - \dfrac{\pi}{3}\right)$

1) بين أن : $A(x) = 2\cos\left(2x - \dfrac{\pi}{6}\right)$

2) حل في المجال $\left]-\pi\,;\pi\right]$ المعادلة : $A(x) = 1$

تمرين4 : ليكن x عددا حقيقيا، نعتبر التعبير : $A(x) = -2\cos^2(x) + \sqrt{3}\sin(2x) + 2$

1) بين أن: $A(x) = 4\sin(x)\left(\dfrac{\sqrt{3}}{2}\cos(x) + \dfrac{1}{2}\sin(x)\right)$

2) حل في IR المعادلة : $A(x) = 0$

تمرين 1 : المعادلات الموجودة بالتمرين كلها على شكل: $a\cos x + b\sin x = c$ ، حيث نعمل و نختزل بـ $r = \sqrt{a^2 + b^2}$

لنحل المعادلة: $\cos x - \sin x = \sqrt{2}$

$$\frac{1}{\sqrt{2}}\cos x - \frac{1}{\sqrt{2}}\sin x = 1$$

$$\cos\left(\frac{\pi}{4}\right)\cos x - \sin\left(\frac{\pi}{4}\right)\sin x = 1$$

لدينا: $\cos\left(\frac{\pi}{4} + x\right) = 1$

بالتالي: $\boxed{S = \left\{\frac{-\pi}{4} + 2k\pi \ / \ k \in Z\right\}}$

$$x + \frac{\pi}{4} = 2k\pi \ / \ k \in Z$$

$$x = \frac{-\pi}{4} + 2k\pi \ / \ k \in Z$$

لنحل المعادلة: $\cos x - \sqrt{3}\sin x = 1$

$$\frac{1}{2}\cos x - \frac{\sqrt{3}}{2}\sin x = \frac{1}{2}$$

$$\cos\left(\frac{\pi}{3}\right)\cos x - \sin\left(\frac{\pi}{3}\right)\sin x = \frac{1}{2}$$

$$\cos\left(\frac{\pi}{3} + x\right) = \frac{1}{2} = \cos\left(\frac{\pi}{3}\right)$$

$$x + \frac{\pi}{3} = \frac{\pi}{3} + 2k\pi \ / \ k \in Z \ \ ou \ \ x + \frac{\pi}{3} = \frac{-\pi}{3} + 2k\pi \ / \ k \in Z$$

$$x = 2k\pi \ / \ k \in Z \ \ ou \ \ x = \frac{-2\pi}{3} + 2k\pi \ / \ k \in Z$$

بالتالي: $\boxed{S = \{2k\pi \ / \ k \in Z\} \cup \left\{\frac{-2\pi}{3} + 2k\pi \ / \ k \in Z\right\}}$

لنحل المعادلة: $\sqrt{3}\cos x - \sin x = \sqrt{2}$

$$\frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\sin x = \frac{\sqrt{2}}{2}$$

$$\cos\left(\frac{\pi}{4}\right)\cos x - \sin\left(\frac{\pi}{4}\right)\sin x = \frac{\sqrt{2}}{2}$$

$$\cos\left(\frac{\pi}{4} + x\right) = \cos\left(\frac{\pi}{4}\right)$$

لدينا:

$$x + \frac{\pi}{4} = \frac{\pi}{3} + 2k\pi \ / \ k \in Z \ \ ou \ \ x + \frac{\pi}{4} = \frac{-\pi}{3} + 2k\pi \ / \ k \in Z$$

$$x = \frac{\pi}{3} - \frac{\pi}{4} + 2k\pi \ / \ k \in Z \ \ ou \ \ x = \frac{-\pi}{3} - \frac{\pi}{4} + 2k\pi \ / \ k \in Z$$

$$x = \frac{\pi}{12} + 2k\pi \ / \ k \in Z \ \ ou \ \ x = \frac{-7\pi}{12} + 2k\pi \ / \ k \in Z$$

بالتالي : $\boxed{S = \left\{\frac{\pi}{12} + 2k\pi \ / \ k \in Z\right\} \cup \left\{\frac{-7\pi}{12} + 2k\pi \ / \ k \in Z\right\}}$

لنحل المعادلة: $\cos\dfrac{x}{2} - \sin\dfrac{x}{2} = -1$

$$\frac{1}{\sqrt{2}}\cos\frac{x}{2} - \frac{1}{\sqrt{2}}\sin\frac{x}{2} = -1$$

$$\cos\left(\frac{\pi}{4}\right)\cos\frac{x}{2} - \sin\left(\frac{\pi}{4}\right)\sin\frac{x}{2} = -1$$

$$\cos\left(\frac{\pi}{4} + \frac{x}{2}\right) = -1$$

لدينا:

$$\frac{x}{2} + \frac{\pi}{4} = \pi + k\pi \;/\; k \in Z$$

$$\frac{x}{2} = \frac{3\pi}{4} + k\pi \;/\; k \in Z$$

$$x = \frac{3\pi}{2} + 2k\pi \;/\; k \in Z$$

بالتالي: $\boxed{S = \left\{ \dfrac{3\pi}{2} + 2k\pi \;/\; k \in Z \right\}}$

لنحل المعادلة: $\sin x + \cos x = \sqrt{2}$

$$\frac{1}{\sqrt{2}}\cos x + \frac{1}{\sqrt{2}}\sin x = 1$$

$$\cos\left(\frac{\pi}{4}\right)\cos x + \sin\left(\frac{\pi}{4}\right)\sin x = 1$$

لدينا: $\cos\left(\dfrac{\pi}{4} - x\right) = 1$

$$x - \frac{\pi}{4} = 2k\pi \;/\; k \in Z$$

$$x = \frac{\pi}{4} + 2k\pi \;/\; k \in Z$$

بالتالي: $\boxed{S = \left\{ \dfrac{\pi}{4} + 2k\pi \;/\; k \in Z \right\}}$

لنحل المعادلة: $\sin x + \cos x = 1$

$$\frac{1}{\sqrt{2}}\cos x + \frac{1}{\sqrt{2}}\sin x = \frac{1}{\sqrt{2}}$$

$$\cos\left(\frac{\pi}{4}\right)\cos x + \sin\left(\frac{\pi}{4}\right)\sin x = \frac{\sqrt{2}}{2}$$

لدينا: $\cos\left(\dfrac{\pi}{4} - x\right) = \dfrac{\sqrt{2}}{2} = \cos\left(\dfrac{\pi}{4}\right)$

$$\frac{\pi}{4} - x = \frac{\pi}{4} + 2k\pi \;/\; k \in Z \; ou \; \frac{\pi}{4} - x = \frac{-\pi}{4} + 2k\pi \;/\; k \in Z$$

$$x = -2k\pi \;/\; k \in Z \; ou \; x = \frac{\pi}{2} + 2k\pi \;/\; k \in Z$$

بالتالي: $\boxed{S = \{ -2k\pi \;/\; k \in Z \} \cup \left\{ \dfrac{\pi}{2} + 2k\pi \;/\; k \in Z \right\}}$

$\cos a = \cos b \Leftrightarrow a = b + 2k\pi \;\; ou \;\; a = -b + 2k\pi \;\;\; / \; k \in Z$

$\cos a = 1 \Leftrightarrow a = 2k\pi \;/\; k \in Z \;\;\; ; \;\; \cos a = -1 \Leftrightarrow a = \pi + 2k\pi \;/\; k \in Z \; ; \; \cos a = 0 \Leftrightarrow a = \dfrac{\pi}{2} + k\pi \;/\; k \in Z$

$\sin a = \sin b \Leftrightarrow \; a = b + 2k\pi \;/\; k \in Z \;\; ou \;\; a = \pi - b + 2k\pi \;/\; k \in Z$

$\sin a = 1 \Leftrightarrow a = \dfrac{\pi}{2} + 2k\pi \;/\; k \in Z \; ; \; \sin a = -1 \Leftrightarrow a = \dfrac{-\pi}{2} + 2k\pi \;/\; k \in Z \; ; \; \sin a = 0 \Leftrightarrow a = k\pi \;/\; k \in Z$

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$$\textbf{تمرين2} : \quad f(x) = \frac{\cos x + \sin x}{\cos x - \sin x}$$

1	لدينا :	لنحل المعادلة: $\sin x - \cos x = 0$

$$\frac{1}{\sqrt{2}}\cos x - \frac{1}{\sqrt{2}}\sin x = 0$$

$$\cos\left(\frac{\pi}{4}\right)\cos x - \sin\left(\frac{\pi}{4}\right)\sin x = 0$$

$$\cos\left(\frac{\pi}{4} + x\right) = 0$$

$$\frac{\pi}{4} + x = \frac{\pi}{2} + k\pi \ / \ k \in Z$$

$$x = \frac{\pi}{4} + k\pi \ / \ k \in Z$$

بالتالي : $\boxed{S = \left\{ \frac{\pi}{4} + k\pi \ / \ k \in Z \right\}}$

2	لدينا : $f(x) = \dfrac{\cos x + \sin x}{\cos x - \sin x}$ إذن حسب السؤال السابق : $Df = \left\{ x \in IR \ / \ \cos x - \sin x \neq 0 \right\} = IR_{-\left\{ \frac{\pi}{4} + k\pi / k \in Z \right\}}$

3	لدينا:

$$f(x) = \frac{(\cos x + \sin x)}{(\cos x - \sin x)} = \frac{(\cos x + \sin x)(\cos x + \sin x)}{(\cos x - \sin x)(\cos x + \sin x)}$$

$$f(x) = \frac{(\cos x + \sin x)^2}{\cos^2 x - \sin^2 x} = \frac{\cos^2 x + \sin^2 x + 2\sin x \cos x}{\cos x \cos x - \sin x \sin x} = \frac{1 + \sin 2x}{\cos(x + x)} = \frac{1 + \sin 2x}{\cos 2x}$$

4	لنحل المعادلة: $f(x) - \sqrt{3} = 0$ ، لدينا :

$$f(x) = \sqrt{3} \Leftrightarrow \frac{1 + \sin 2x}{\cos 2x} = \sqrt{3} \Leftrightarrow 1 + \sin 2x = \sqrt{3}\cos 2x \Leftrightarrow \sqrt{3}\cos 2x - \sin 2x = 1$$

$$\Leftrightarrow \frac{\sqrt{3}}{2}\cos 2x - \frac{1}{2}\sin 2x = \frac{1}{2} \Leftrightarrow \cos\left(\frac{\pi}{6}\right)\cos 2x - \sin\left(\frac{\pi}{6}\right)\sin 2x = \frac{1}{2}$$

$$\Leftrightarrow \cos\left(\frac{\pi}{6} + 2x\right) = \cos\left(\frac{\pi}{3}\right)$$

$$\Leftrightarrow \frac{\pi}{6} + 2x = \frac{\pi}{3} + 2k\pi \ / \ k \in Z \quad ou \quad \frac{\pi}{6} + 2x = \frac{-\pi}{3} + 2k\pi \ / \ k \in Z$$

$$\Leftrightarrow 2x = \frac{\pi}{6} + 2k\pi \ / \ k \in Z \quad ou \quad 2x = \frac{-\pi}{2} + 2k\pi \ / \ k \in Z$$

$$f(x) = \sqrt{3} \Leftrightarrow x = \frac{\pi}{12} + k\pi \ / \ k \in Z \quad ou \quad x = \frac{-\pi}{4} + k\pi \ / \ k \in Z$$

بالتالي: $\boxed{S = \left\{ \frac{\pi}{12} + k\pi \ / \ k \in Z \right\} \cup \left\{ \frac{-\pi}{4} + k\pi \ / \ k \in Z \right\}}$

$$\textbf{تمرين3} : \quad A(x) = \sqrt{3}\cos\left(2x - \frac{\pi}{3}\right) - \sin\left(2x - \frac{\pi}{3}\right)$$

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$$A(x) = \sqrt{3}\cos\left(2x - \frac{\pi}{3}\right) - \sin\left(2x - \frac{\pi}{3}\right) = 2\left(\frac{\sqrt{3}}{2}\cos\left(2x - \frac{\pi}{3}\right) - \frac{1}{2}\sin\left(2x - \frac{\pi}{3}\right)\right)$$

$$= 2\left(\cos\left(\frac{\pi}{6}\right)\cos\left(2x - \frac{\pi}{3}\right) - \sin\left(\frac{\pi}{6}\right)\sin\left(2x - \frac{\pi}{3}\right)\right)$$

$$A(x) = 2\cos\left(\frac{\pi}{6} + 2x - \frac{\pi}{3}\right) = 2\cos\left(2x - \frac{\pi}{6}\right)$$

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$$A(x) = 1 \Leftrightarrow 2\cos\left(2x - \frac{\pi}{6}\right) = 1 \Leftrightarrow \cos\left(2x - \frac{\pi}{6}\right) = \frac{1}{2} = \cos\left(\frac{\pi}{3}\right)$$

$$\Leftrightarrow 2x - \frac{\pi}{6} = \frac{\pi}{3} + 2k\pi / k \in Z \quad ou \quad 2x - \frac{\pi}{6} = \frac{-\pi}{3} + 2k\pi / k \in Z$$

$$\Leftrightarrow 2x = \frac{\pi}{2} + 2k\pi / k \in Z \quad ou \quad 2x = \frac{-\pi}{6} + 2k\pi / k \in Z$$

$$A(x) = 1 \Leftrightarrow x = \frac{\pi}{4} + k\pi / k \in Z \quad ou \quad x = \frac{-\pi}{12} + k\pi / k \in Z$$

لتحديد الحلول الموجودة في المجال $] -\pi ; \pi]$ نؤطر العدد النسبي k و نحدد قيمته:

$$\frac{\pi}{4} + k\pi \in] -\pi ; \pi] \Leftrightarrow -\pi < \frac{\pi}{4} + k\pi \leq \pi \Leftrightarrow -4\pi < \pi + 4k\pi \leq 4\pi \Leftrightarrow -5\pi < 4k\pi \leq 3\pi$$

$$\Leftrightarrow -5 < 4k \leq 3 \quad \Leftrightarrow \frac{-5}{4} < k \leq \frac{3}{4} \Leftrightarrow k = -1 \ ou \ k = 0$$

$$\frac{\pi}{4} + k\pi \in] -\pi ; \pi] \Leftrightarrow x = \frac{\pi}{4} - \pi = \frac{-3\pi}{4} \quad ou \quad x = \frac{\pi}{4}$$

$$\frac{-\pi}{12} + k\pi \in] -\pi ; \pi] \Leftrightarrow -\pi < \frac{-\pi}{12} + k\pi \leq \pi \Leftrightarrow -12\pi < -\pi + 12k\pi \leq 12\pi \Leftrightarrow -11\pi < 12k\pi \leq 13\pi$$

$$\Leftrightarrow -11 < 12k \leq 13 \quad \Leftrightarrow \frac{-11}{12} < k \leq \frac{13}{12} \Leftrightarrow k = 0 \ ou \ k = 1 \qquad : و$$

$$\frac{-\pi}{12} + k\pi \in] -\pi ; \pi] \Leftrightarrow x = \frac{-\pi}{12} \quad ou \quad x = \frac{-\pi}{12} + \pi = \frac{11\pi}{12}$$

بالتالي: $\boxed{S = \left\{ \frac{\pi}{4} ; \frac{-3\pi}{4} ; \frac{-\pi}{12} ; \frac{11\pi}{12} \right\}}$

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تمرين4 : $\quad A(x) = -2\cos^2(x) + \sqrt{3}\sin(2x) + 2$

لدينا:
$$A(x) = -2\cos^2(x) + \sqrt{3}\sin(2x) + 2 = 2(1 - \cos^2 x) + \sqrt{3} \times 2\sin x \cos x$$

$$= 2\sin^2 x + 2\sqrt{3}\sin x \cos x = 4\sin x \left(\frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x\right)$$

$$A(x) = 4\sin(x)\left(\frac{\sqrt{3}}{2}\cos(x) + \frac{1}{2}\sin(x)\right)$$

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$$A(x) = 0 \Leftrightarrow 4\sin(x)\left(\frac{\sqrt{3}}{2}\cos(x) + \frac{1}{2}\sin(x)\right) = 0$$

$$\Leftrightarrow \sin(x) = 0 \quad ou \quad \left(\frac{\sqrt{3}}{2}\cos(x) + \frac{1}{2}\sin(x)\right) = 0$$

$$\Leftrightarrow x = k\pi / k \in Z \quad ou \quad \cos\left(\frac{\pi}{6}\right)\cos x + \sin\left(\frac{\pi}{6}\right)\sin x = 0$$

$$\Leftrightarrow x = k\pi / k \in Z \quad ou \quad \cos\left(x - \frac{\pi}{6}\right) = 0$$

$$\Leftrightarrow x = k\pi / k \in Z \quad ou \quad x - \frac{\pi}{6} = \frac{\pi}{2} + k\pi / k \in Z$$

$$A(x) = 0 \Leftrightarrow x = k\pi / k \in Z \quad ou \quad x = \frac{2\pi}{3} + k\pi / k \in Z$$

بالتالي: $\boxed{S = \{k\pi / k \in Z\} \cup \left\{\frac{2\pi}{3} + k\pi / k \in Z\right\}}$

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