المملكة المغربية
وزارة التربية الوطنية
والتكوين المهني
والتعليم العالي والبحث العلمي

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المركز الوطني للتقويم والامتحانات والتوجيه

الامتحان الوطني الموحد للبكالوريا
المسالك الدولية - خيار إنجليزية
الدورة العادية 2017
- الموضوع -

NS 24E

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4	مدة الإنجاز	الرياضيات	المادة
9	المعامل	مسلك العلوم الرياضية (أ) و(ب) – خيار إنجليزية	الشعبة أو المسلك

- The duration of the exam is 4 hours

- The exam contains 4 independent exercises

- The exercises can be treated in the order chosen by the candidate

- The exercise1 concerns the algebraic structures.......(3,5pts)

-The exercise2 concerns the complex numbers…........(3,5pts)

- The exercise3 concerns the arithmetic……………………(3pts)

-The exercise4 concerns the analysis……………..………..(10pts)

The use of calculators is not allowed

The use of red pens on answers sheets is not permitted

EXERCISE1 : (3.5 points)

Recall that $(M_3(\mathbb{R}),+,\times)$ is a ring with unity $I=\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}$, its additive identity is the null matrix $O=\begin{pmatrix}0&0&0\\0&0&0\\0&0&0\end{pmatrix}$, and that $(\mathbb{C},+,\times)$ is a commutative field.

Put $A=\begin{pmatrix}1&0&0\\1&1&0\\1&1&1\end{pmatrix}$ and $M(a,b)=\begin{pmatrix}a&b&-b\\0&0&0\\b&-a&a\end{pmatrix}$ for all (a,b) in $\mathbb{R}^2$.

Consider the set $E=\{M(a,b)/(a,b)\in\mathbb{R}^2\}$

0.5 1-Show that E is a sub-group of the group $(M_3(\mathbb{R}),+)$

0.5 2- We define in $M_3(\mathbb{R})$ the internal composition law $\top$ by :

$$\forall(a,b,c,d)\in\mathbb{R}^4 \qquad M(a,b)\top M(c,d)=M(a,b)\times A\times M(c,d)$$

Verify that E is stable in $(M_3(\mathbb{R}),\top)$

3- Let φ be the mapping from $\mathbb{C}^*$ to E which associates to every non zero complex number $a+ib$ (where $(a,b)\in\mathbb{R}^2$) the matrix $M(a,b)$ in E

0.75 a) Verify that φ is an homomorphism from $(\mathbb{C}^*,\times)$ to $(E,\top)$ and that $\varphi(\mathbb{C}^*)=E^*$ where $E^*=E\setminus\{M(0,0)\}$

0.75 b) Deduce that $(E^*,\top)$ is a commutative group which neutral element J must be determined

0.5 4- a) Show that the inner composition law "$\top$" is distributive on the inner composition law "+" in E

0.5 b) Deduce that $(E,+,\top)$ is a commutative field.

EXERCISE2 :(3.5 points)

Let m be a **non zero** complex number.

Part1 :

Consider in $\mathbb{C}$ the following equation :

$$(E):\quad 2z^2-2(m+1+i)z+m^2+(1+i)m+i=0$$

0.5 1- Verify that the discriminant of the equation (E) is $\Delta=(2im)^2$

0.5 2- Solve in $\mathbb{C}$ the equation (E)

Part2 :

The complex plane is attached to a direct orthonormal coordinate system $(O,\vec{e_1},\vec{e_2})$

Assume that $m \in \mathbb{C} - \{0,1,i\}$ and put : $z_1 = \frac{1+i}{2}(m+1)$ and $z_2 = \frac{1-i}{2}(m+i)$

Consider the points A, B, M, M_1 and M_2, which affixes are respectively 1, i, m, z_1 and z_2

0.25 1-a) Verify that : $z_1 = iz_2 + 1$

0.5 b) Show that M_1 is the image of M_2 under the rotation having center the point Ω with affix $\omega = \frac{1+i}{2}$ and angle with measurement $\frac{\pi}{2}$

0.5 2-a) Verify that : $\frac{z_2 - m}{z_1 - m} = i\frac{m-1}{m-i}$

0.5 b) Show that if the points M, M_1 and M_2 are collinear then M belongs to the circle (Γ) with diameter $[AB]$

0.75 c) Determine the set of the points M for which the points Ω, M, M_1 and M_2 are cocyclic (note that: $\frac{z_1 - \omega}{z_2 - \omega} = i$).

EXERCISE3 :(3points)

We admit that 2017 is a prime number and that $2016 = 2^5 3^2 7$

Let p be a **prime number greater or equal than 5**

1- Let (x,y) in $\mathbb{N}^* \times \mathbb{N}^*$ such that: $px + y^{p-1} = 2017$

0.25 a) Verify that: $p < 2017$

0.5 b) Show that: p does not divides y

0.75 c) Show that : $y^{p-1} \equiv 1 \ [p]$ and then deduce that p divides 2016

0.5 d) Show that : $p = 7$

1 2- Determine , according the values of p, the couples of $\mathbb{N}^* \times \mathbb{N}^*$ verifying :

$$px + y^{p-1} = 2017$$

EXERCISE4 :(10 points)

Part1 :

Consider the numerical function f defined on $[0,+\infty[$ by :

$$f(0)=0 \text{ and } (\forall x\in]0,+\infty[) \quad f(x)=\left(1+\frac{1}{x}\right)e^{-\frac{1}{x}}$$

Let (C) be the representative curve of f in an orthonormal coordinate system $(O,\vec{i},\vec{j})$ (We take : $\|\vec{i}\|=\|\vec{j}\|=2cm$)

0.25 1-a) Show that the function f is continuous on the right at the point 0

0.5 b) Show that the function f is differentiable on the right at the point 0

0.5 c) Show that the function f is differentiable on $]0,+\infty[$ and calculate $f'(x)$ for all x in the interval $]0,+\infty[$

0.5 2- a) Calculate $\lim_{x\to+\infty} f(x)$ and then interpret graphically the obtained result.

0.25 b) Give the variations table of the function f

0.75 3- a) Show that the curve (C) admits an inflection point I which coordinates must be determined

0.5 b) Plot the curve (C) (we take : $f(1);\ 0,7$ and $4e^{-3};\ 0,2$)

Part2 :

Consider the numerical function F defined on $[0,+\infty[$ by :

$$F(x)=\int_x^1 f(t)\,dt$$

0.25 1-Show that the function F is continuous over $[0,+\infty[$

0.5 2-a) By using the method of integration by parts show that :

$$(\forall x\in]0,+\infty[) \quad \int_x^1 e^{-\frac{1}{t}}dt = e^{-1} - xe^{-\frac{1}{x}} - \int_x^1 \frac{1}{t}e^{-\frac{1}{t}}dt$$

0.25 b) Determine $\int_x^1\left(1+\frac{1}{t}\right)e^{-\frac{1}{t}}dt$ for all x in $]0,+\infty[$

0.5 c) Show that: $\int_0^1 f(t)\,dt=e^{-1}$

0.5 3-Calculate in square centimeter (cm^2), the area of the region of the plane enclosed by the curve (C) and the lines with the equations: $x=0$, $x=2$ et $y=0$.

4-Consider the numerical sequence $(u_n)_{n\geq0}$ defined by : $u_n=F(n)-F(n+2)$

0.5 a) By using the mean value theorem , show that for every natural number n

	there exist a real number v_n in the interval $]n, n+2[$ such that: $u_n = 2\left(1+\frac{1}{v_n}\right)e^{-\frac{1}{v_n}}$
0.25	b) Show that : $(\forall n \in \mathbb{N}^*)$ $\quad 2\left(1+\frac{1}{n}\right)e^{-\frac{1}{n}} \leq u_n \leq 2\left(1+\frac{1}{n+2}\right)e^{-\frac{1}{n+2}}$
0.25	c) Deduce $\lim\limits_{n\to+\infty} u_n$
	<u>Part3</u> :
0.5	1-a) Show that for all non zero natural number n, there exist a unique non zero positive real number a_n such that : $f(a_n) = e^{-\frac{1}{n}}$
0.25	b) Show that the numerical sequence $(a_n)_{n\geq 1}$ is increasing.
0.25	c) Verify that: $(\forall n \in IN^*)$ $\quad -\frac{1}{a_n} + \ln\left(1+\frac{1}{a_n}\right) = -\frac{1}{n}$
0.25	2-a) Show that: $(\forall t \in [0,+\infty[)$ $\quad 1-t \leq \frac{1}{1+t} \leq 1-t+t^2$
0.5	b) Show that: $(\forall x \in [0,+\infty[)$ $\quad -\frac{x^2}{2} \leq -x+\ln(1+x) \leq -\frac{x^2}{2}+\frac{x^3}{3}$
	3- Let n be a natural number greater or equal than 4.
0.5	a) Verify that: $a_4 \geq 1$ and then deduce that: $a_n \geq 1$ (we admit that $e^{\frac{3}{4}} \geq 2$)
0.5	b) Show that: $1-\frac{2}{3a_n} \leq \frac{2a_n^{\,2}}{n} \leq 1$ (one can use the questions 1-c) and 2-b) of Part 3)
0.5	c) Show that: $\sqrt{\frac{n}{6}} \leq a_n$ (one can use the questions 3-a) and 3-b)) and then deduce $\lim\limits_{n\to+\infty} a_n$
0.5	d) Determine : $\lim\limits_{n\to+\infty} a_n\sqrt{\frac{2}{n}}$

END

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المركز الوطني للتقويم والامتحانات والتوجيه

الامتحان الوطني الموحد للبكالوريا
المسالك الدولية - خيار إنجليزية
الدورة العادية 2017
- عناصر الإجابة -

NR 24E

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المادة	الرياضيات	مدة الإنجاز	4
الشعبة أو المسلك	مسلك العلوم الرياضية (أ) و(ب) – خيار إنجليزية	المعامل	9

EXERCISE1		Indication of solutions	Barème
1-		E is a subgroup of $(M_3(\mathbb{R}),+)$	0.5
2-		stability	0.5
3-	a)	homomorphism……0.5 $\varphi(\mathbb{C}^*)=E^*$ …… 0.25	0.75
	b)	$(E^*,\top)$ is a commutative group……0.5 the determination of J …… ….0.25	0.75
4-	a)	distributivity	0.5
	b)	$(E,+,\top)$ commutative field	0.5

EXERCISE2			Indication of solutions	Barème
Part1	1-		verification of Δ	0.5
	2-		The solutions of the equation	0.5
Part2	1-	a)	verification of the relation : $z_1 = iz_2 + 1$	0.25
		b)	M_1 is the image of M_2	0.5
	2-	a)	verification of the equality	0.5
		b)	the belong to (Γ)	0.5
		c)	The line (AB) deprivited of the two points A and B the over all marcks is assigned even if the candidate does not treated the case of the points A and B	0.75

EXERCISE3		Indication de solutions	Barème
1-	a)	verification	0.25
	b)	p does not divides y	0.5
	c)	application of the FERMAT theorem……0.25 deduction……0.5	0.75

	d)	$p=7$	0.5
2-		If $p \neq 7$ there is no couple.......................0.25 If $p=7$ The couples are : $(288,1)$ and $(279,2)$ and $(184,3)$...0.75	1

EXERCISE4			Indication de solutions	Barème
Part1	1-	a)	the continuity on the right at 0	0.25
		b)	the differenciability on the right at 0	0.5
		c)	- differenciability on $]0,+\infty[$................ 0 .25 - calculus of de $f'(x)$........................... 0.25	0.5
	2-	a)	- $\lim\limits_{x\to+\infty} f(x)=1$................. 0.25 - graphical interpretation 0.25	0.5
		b)	variation table	0.25
	3-	a)	- calculus de $f''(x)$ .... 0.25 -signe of $f''(x)$ .... 0 .25 - inflexion point $I\left(\frac{1}{3},4e^{-3}\right)$................0.25	0.75
		b)	the plot of (C)	0.5
Part2	1-		the continuity of F on $[0,+\infty[$	0.25
	2-	a)	the integration by parts	0.5
		b)	determination of the integral in terms of en x : $\int_x^1\left(1+\frac{1}{t}\right)e^{-\frac{1}{t}}dt=e^{-1}-xe^{-\frac{1}{x}}$	0.25
		c)	determination of the value of the integral : the function F is continuous on the right at 0 then : $\int_0^1 f(t)dt=F(0)=\lim\limits_{x\to0^+}F(x)=\lim\limits_{x\to0^+}\int_x^1 f(t)dt=e^{-1}$	0.5

	3-		calculus of the air (in squarre centimeter): $A = \left(\int_0^1 f(t)dt + \int_1^2 f(t)dt\right) \times 4cm^2$	0.5
	4-	a)	the equality	0.5
		b)	the double inequality	0.25
		c)	$\lim_{n \to +\infty} u_n = 2$	0.25
Part3	1-	a)	existence , iniquity de a_n et $a_n > 0$	0.5
		b)	the sequence $(a_n)_{n \geq 1}$ is increasing	0.25
		c)	verification of the equality	0.25
	2-	a)	the double inequality	0.25
		b)	the double inequality	0.25
	3-	a)	verification……………………………………………0.25 deduction……………………………………………0.25	0.5
		b)	the double inequality	0.5
		c)	inequality ………………………………………………0.25 deduction………………………………………………0.25	0.5
		d)	determination of the limit	0.5