الامتحان الوطني الموحد للبكالوريا

المسالك الدولية – خيار أنجليزية

الدورة العادية 2018

-عناصر الإجابة-

NR 24E

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المملكة المغربية
وزارة التربية الوطنية
والتكوين المهني
والتعليم العالي والبحث العلمي

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المركز الوطني للتقويم والامتحانات والتوجيه

4	مدة الإنجاز	الرياضيات	المادة
9	المعامل	شعبة العلوم الرياضية : " أ" و " ب" – خيار أنجليزية	الشعبة أو المسلك

Exercise1		Indications of solutions	M
1-		E is a sub-group of $\left(M_2(\square),+\right)$	0.25
2-	a)	E is a sub vector space	0.25
	b)	(I,J) is free..........025 (I,J) is generator......0.25	0.5
3-	a)	E is stable	0.5
	b)	$(E,+,\times)$ is a commutaive ring	0.5
4-	a)	φ morphism	0.5
	b)	$\varphi(\square^*)=E^*$	0.5
	c)	$(E^*,\times)$ commutative group	0.25
5-		$(E,+,\times)$ commutative field	0.25

Exercise2		Indications of solutions	M
1-		the implication ($p-5$ is even)	0.5
2-	a)	$x\wedge p=1$ (BEZOUT ‘s theorem or every other correct method)	0.5
	b)	$x^{p-1}\equiv 1\ [p]$ (FERMAT ‘s theorem or every other correct method)	0.5

	c)	verification of the equality	0.5
	d)	deduction : $x^2.x^{(k-1)(p-1)} \equiv x^{k(p-5)}$ $[p]$	0.5
3-		67 is prime and $67 = 3 + 4 \times 16$..........0.25 Application of results of exercise..........0.25	0.5

Exercise3			Indications of solutions	M
I-	1-	a)	verification	0.25
		b)	Case of $m = 2$ $S = \{-1-i\}$ Case of $m \neq 2$ $S = \{-1-i, -1+i-im\}$	0.5
	2-		- exponential Polar form of the first solution ....0.25 - exponential Polar form of the second solution 0.25	0.5
II-	1-	a)	verification	0.25
		b)	$b = 2$	0.5
	2-	a)	Verification of the equality	0.5
		b)	The equivalence : 0.25 for each implication The solution is accepted even if the case M=A is not treated	0.5
		c)	Determination of the circle ....0.25 Determination of radius and center.......0.25 The solution is accepted even if the case M=A is not treated	0.5

Exercise4			Indications of solutions	M
Part I	1-	a)	The equality	0.5
		b)	The equality	0.5
		c)	0.25 for each inequality	0.5

	2-		Calculus of the limit	0.25
Part II	1-	a)	Continuity on right at O	0.25
		b)	Differentiability on right at 0..................0.25	0.5
		c)	Calculus of $\lim\limits_{x\to+\infty} f(x)$ 0.25	0.75
			Calculus of $\lim\limits_{x\to+\infty} \frac{f(x)}{x}$ 0.25	
			graphical Interpretation 0.25	
	2-	a)	Differentiability.......................................0.25	0.5
			Calculus of $f'(x)$.........0.25	
		b)	f is monotonic	0.25
		c)	verification	0.25
	3-		graphic representation..............................0.25 the right half-tangent at the point 0.........0.25	0.5
Part III	1-	a)	0.25 for each inequality	0.5
		b)	g is monotonic 0.25	0.5
			The image of $]0,+\infty[$ by g 0.25	
		c)	Existence and uniquity of α	0.25
	2-	a)	Sign of the sequence	0.25
		b)	MVT (Mean values theorem)	0.25
		c)	induction	0.5
		d)	Convergence of the sequence ; $\lim\limits_{n\to\infty} u_n = \alpha$	0.5

Exercise5	Indications of solutions	M
1-	F is contnious on $\mathbb{R}$ 0.25	0.5

		F is strictly increasing on $\mathbb{R}$ ……………..0.25	
2-	a)	The inequity $F(x) \geq x$ ……………0.25	0.5
		Calculus of $\lim\limits_{x \to +\infty} F(x)$ ……………..0.25	
	b)	F is odd (any other correct solution is accepted) ……………..0.25	0.5
		Calculus of $\lim\limits_{x \to -\infty} F(x)$ ……………..0.25	
	c)	F is a bijection from $\mathbb{R}$ to $\mathbb{R}$	0.5
	d)	Differentiability of G at 0……………..0.25	0.5
		Calculus of $G'(0)$ ……………………….0.25	

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الامتحان الوطني الموحد للبكالوريا

المسالك الدولية – خيار أنجليزية

الدورة العادية 2018

-الموضوع-

NS 24E

المادة	الرياضيات	مدة الإنجاز	4
الشعبة أو المسلك	شعبة العلوم الرياضية : " أ" و " ب" – خيار أنجليزية	المعامل	9

- The duration of the exam is 4 hours.

- The exam contains 5 independent exercises.

- The exercises can be treated in the order chosen by the candidate.

- Exercise 1 concerns the algebraic structures.................(3.5pts)

- Exercise 2 concerns the arithmetic ....(3 pts)

- Exercise 3 concerns the complex numbers ....(3.5pts)

- Exercise 4 concerns the analysis....................................(7.5pts)

- Exercise 5 concerns the analysis....................................(2.5pts)

The use of calculators is not allowed

The use of red pens on answers sheets is not permitted

EXERCISE 1 :(3.5 points)

We recall that $(\mathbb{C},+,\times)$ is a commutative field and that $(M_2(\mathbb{R}),+,.)$ is a real vector space and that $(M_2(\mathbb{R}),+,\times)$ is a commutative ring with unity $I=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$, its additive identity is the null matrix $O=\begin{pmatrix}0 & 0\\ 0 & 0\end{pmatrix}$

For every $(x,y)\in\mathbb{R}^2$ we put $M(x,y)=\begin{pmatrix}x & -2y\\ y & x+2y\end{pmatrix}$

and we consider the set $E=\{M(x,y)/(x,y)\in\mathbb{R}^2\}$

0.25 1- Show that E is a sub-group of the group $(M_2(\mathbb{R}),+)$

0.25 2- a) Show that E is a sub-space of the real vector space $(M_2(\mathbb{R}),+,.)$

0.5 b) We put $J=M(0,1)$. Show that (I,J) is a basis of the real vector space $(E,+,.)$

0.5 3-a) Show that E is stable in $(M_2(\mathbb{R}),\times)$

0.5 b) Show that $(E,+,\times)$ is a commutative ring.

4- Let φ be the mapping from $\mathbb{C}^*$ to $M_2(\mathbb{R})$ defined by :

$$\left(\forall(x,y)\in\mathbb{R}^2-\{(0,0)\}\right)\quad;\quad \varphi(x+iy)=M(x+y,-y)=\begin{pmatrix}x+y & 2y\\ -y & x-y\end{pmatrix}$$

0.5 a) Show that φ is an homomorphism from $(\mathbb{C}^*,\times)$ to $(M_2(\mathbb{R}),\times)$

0.5 b) We put $E^*=E-\{O\}$. Show that : $\varphi(\mathbb{C}^*)=E^*$

0.25 c) Deduce that $(E^*,\times)$ is a commutative group.

0.25 5- Show that $(E,+,\times)$ is a commutative field .

EXERCISE 2: (3 points)

Let p be a prime number such that $p=3+4k$ $\quad(k\in\mathbb{N}^*)$

0.5 1- Show that for all integer number x, if $x^2\equiv 1\ [p]$ then $x^{p-5}\equiv 1\ [p]$

2- Let x be an integer number verifying : $x^{p-5}\equiv 1\ [p]$

0.5 a) Show that x and p are coprime.

0.5 b) Show that : $x^{p-1} \equiv 1 \ [p]$

0.5 c) Verify that : $2+(k-1)(p-1) = k(p-5)$

0.5 d) Deduce that : $x^2 \equiv 1 \ [p]$

0.5 3) Solve in $\mathbb{Z}$ the equation : $x^{62} \equiv 1 \ [67]$

EXERCISE 3 : (3.5 points)

Let m be a complex number.

I- We consider in the set of the complex numbers $\mathbb{C}$ the equation (E_m) with variable z :

$$z^2 + (im+2)z + im + 2 - m = 0$$

0.25 1-a) Verify that $\Delta = (im - 2i)^2$ is the discriminant of the equation (E_m)

0.5 b) Give according the values of m , the set of the solutions of the equation (E_m)

0.5 2- For $m = i\sqrt{2}$, rewrite the two solutions of the equation (E_m) in exponential polar form.

II- The complex plane is attached to a direct orthonormal coordinate system $(O;\vec{u},\vec{v})$

We consider the points A , Ω , M and M', with respective affixes $a = -1-i$, $\omega = i$, m and $m' = -im - 1 + i$

1- Let R be the rotation with angle $-\frac{\pi}{2}$ which send M to M'

0.25 a) Verify that Ω is the center of the rotation R

0.5 b) Determine the affix b of B, where B is the point verifying : $A = R(B)$

0.5 2-a) Verify that : $m' - a = \frac{\omega - a}{\omega - b}(m - b)$

0.5 b) Deduce that the points A , M and M' are collinear if and only if the points A , B, Ω and M are cocyclic.

0.5 c) Show that the set of the points M such that the points A , M and M' are collinear is a circle which center and radius are to be determined.

EXERCISE 4 :(7.5 points)

PART I

0.5 1-a) Show that : $(\forall x \in]0,+\infty[)$; $\int_0^x \frac{t}{1+t}dt = x - \ln(1+x)$

b) By using the change of variable $u = t^2$, show that :

0.5 $$(\forall x \in]0,+\infty[); \quad \int_0^x \frac{t}{1+t}dt = \frac{1}{2}\int_0^{x^2} \frac{1}{1+\sqrt{u}}du$$

0.5 c) Deduce that : $\left(\forall x\in\left]0,+\infty\right[\right)$; $\dfrac{1}{2(1+x)}\le\dfrac{x-\ln(1+x)}{x^2}\le\dfrac{1}{2}$

0.25 2- Determine : $\lim\limits_{x\to0^+}\dfrac{x-\ln(1+x)}{x^2}$

PART II

We consider the numerical function f defined on $\left[0,+\infty\right[$ by :

$$\begin{cases} f(x)=\left(\dfrac{x+1}{x}\right)\ln(1+x) & ;\ x\neq0 \\ f(0)=1 \end{cases}$$

and let (C) be the representative curve of f in an orthonormal coordinate system $(O;\vec{i},\vec{j})$

0.25 1-a)Show that f is right-continuous at the point 0

0.5 b) Show that f is differentiable on the right at the point 0

(One can use the result of the question I-2)

0.75 c) Compute : $\lim\limits_{x\to+\infty}f(x)$, $\lim\limits_{x\to+\infty}\dfrac{f(x)}{x}$ and interpret graphically the obtained result.

0.5 2- a) Show that f is differentiable on $\left]0,+\infty\right[$, and verify that :

$$\left(\forall x\in\left]0,+\infty\right[\right)\quad;\quad f'(x)=\frac{x-\ln(1+x)}{x^2}$$

0.25 b) Deduce that f is strictly increasing on $\left[0,+\infty\right[$

0.25 c) Verify that : $f\left(\left[0,+\infty\right[\right)=\left[1,+\infty\right[$

0.5 3- Plot the curve (C) (the right half-tangent at the point 0 must be ploted)

PART III

1- Let g be the numerical function defined on $\left]0,+\infty\right[$ by: $g(x)=f(x)-x$

0.5 a) Show that : $\left(\forall x\in\left]0,+\infty\right[\right)$; $0\le f'(x)\le\dfrac{1}{2}$

0.5 b) Deduce that g is strictly decreasing on $\left]0,+\infty\right[$ and show that $g\left(\left]0,+\infty\right[\right)=\left]-\infty,1\right[$

0.25 c) Show that the equation $f(x)=x$ admits a unique solution α in $\left]0,+\infty\right[$

2- Let a be a real number in $\left]0,+\infty\right[$.

Consider the sequence $(u_n)_{n\ge0}$ defined by : $u_0=a$ and $(\forall n\in\mathbb{N})$; $u_{n+1}=f(u_n)$

0.25 a) Demonstrate that : $(\forall n\in\mathbb{N})$; $u_n>0$

0.5 b) Show that : $(\forall n\in\mathbb{N})$; $\left|u_{n+1}-\alpha\right|\le\dfrac{1}{2}\left|u_n-\alpha\right|$

0.5 c) Prove by induction that : $(\forall n \in \mathbb{N})$; $|u_n - \alpha| \leq \left(\frac{1}{2}\right)^n |a - \alpha|$

0.25 d) Show that the sequence $(u_n)_{n\geq 0}$ converge to α

EXERCISE 5 : (2.5 points)

Consider the numerical function F defined on $\mathbb{R}$ by: $F(x) = \int_0^x e^{t^2} dt$

0.5 1- Show that F is continuous and is strictly increasing on $\mathbb{R}$

0.5 2-a) Show that : $(\forall x \in]0,+\infty[)$; $F(x) \geq x$, and then deduce $\lim_{x\to+\infty} F(x)$

0.5 b) Show that F is odd , and then deduce $\lim_{x\to-\infty} F(x)$

0.5 c) Show that F is a bijection from $\mathbb{R}$ to $\mathbb{R}$

0.5 d) Show that the inverse bijection G of the function F is differentiable at 0 and compute $G'(0)$

END