

الامتحان الوطني الموحد للبكالوريا  
المسالك الدولية – خيار إنجليزية  
الدورة العادية 2018  
-عناصر الإجابة-

NR 24E

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المملكة المغربية  
وزارة التربية الوطنية  
والتكوين المهني  
والتعليم العالي والبحث العلمي

المركز الوطني للتقويم والامتحانات  
والتوجيه



4	مدة الإنجاز	الرياضيات	المادة
9	المعامل	شعبة العلوم الرياضية : " أ " و " ب " - خيار إنجليزية	الشعبة أو المسلك

Exercise1	Indications of solutions		M
1-	$E$ is a sub-group of $(M_2(\square), +)$		0.25
2-	a)	$E$ is a sub vector space	0.25
	b)	$(I, J)$ is free.....0.25 $(I, J)$ is generator.....0.25	0.5
3-	a)	$E$ is stable	0.5
	b)	$(E, +, \times)$ is a commutative ring	0.5
4-	a)	$\varphi$ morphism	0.5
	b)	$\varphi(\square^*) = E^*$	0.5
	c)	$(E^*, \times)$ commutative group	0.25
5-	$(E, +, \times)$ commutative field		0.25

Exercise2	Indications of solutions		M
1-	the implication ( $p - 5$ is even)		0.5
2-	a)	$x \wedge p = 1$ ( BEZOUT 's theorem or every other correct method )	0.5
	b)	$x^{p-1} \equiv 1 [p]$ ( FERMAT 's theorem or every other correct method)	0.5

	c)	verification of the equality	0.5
	d)	deduction : $x^2 \cdot x^{(k-1)(p-1)} \equiv x^{k(p-5)} \quad [p]$	0.5
3-		67 is prime and $67 = 3 + 4 \times 16 \dots\dots\dots 0.25$ Application of results of exercise.....0.25	0.5

Exercise3			Indications of solutions	M
I-	1-	a)	verification	0.25
		b)	Case of $m = 2 \quad S = \{-1 - i\}$ Case of $m \neq 2 \quad S = \{-1 - i, -1 + i - im\}$	0.5
	2-		- exponential Polar form of the first solution .....0.25	0.5
			- exponential Polar form of the second solution .....0.25	
II-	1-	a)	verification	0.25
		b)	$b = 2$	0.5
	2-	a)	Verification of the equality	0.5
		b)	The equivalence : 0.25 for each implication The solution is accepted even if the case M=A is not treated	0.5
		c)	Determination of the circle .....0.25 Determination of radius and center.....0.25 The solution is accepted even if the case M=A is not treated	0.5

Exercise4		Indications of solutions	M	
Part I	1-	a)	The equality	0.5
		b)	The equality	0.5
		c)	0.25 for each inequality	0.5



	2-		Calculus of the limit	0.25	
Part II	1-	a)	Continuity on right at 0	0.25	
		b)	Differentiability on right at 0.....0.25	0.5	
		c)	Calculus of $\lim_{x \rightarrow +\infty} f(x)$ .....0.25	0.75	
		Calculus of $\lim_{x \rightarrow +\infty} \frac{f(x)}{x}$ .....0.25			
		graphical Interpretation .....0.25			
	2-	a)		Differentiability.....0.25	0.5
				Calculus of $f'(x)$ .....0.25	
		b)	$f$ is monotonic	0.25	
		c)	verification	0.25	
	3-		graphic representation.....0.25	0.5	
		the right half-tangent at the point 0.....0.25			
Part III	1-	a)	0.25 for each inequality	0.5	
		b)	$g$ is monotonic .....0.25	0.5	
			The image of $]0, +\infty[$ by $g$ .....0.25		
		c)	Existence and unicity of $\alpha$	0.25	
	2-	a)	Sign of the sequence	0.25	
		b)	MVT ( Mean values theorem)	0.25	
		c)	induction	0.5	
		d)	Convergence of the sequence ; $\lim_{n \rightarrow \infty} u_n = \alpha$	0.5	

Exercise5	Indications of solutions	M
1-	$F$ is contnious on $\square$ .....0.25	0.5



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		$F$ is strictly increasing on $\mathbb{R}$ .....0.25	
2-	a)	The inequity $F(x) \geq x$ .....0.25	0.5
		Calculus of $\lim_{x \rightarrow +\infty} F(x)$ .....0.25	
	b)	$F$ is odd (any other correct solution is accepted ) .....0.25	0.5
		Calculus of $\lim_{x \rightarrow -\infty} F(x)$ .....0.25	
	c)	$F$ is a bijection from $\mathbb{R}$ to $\mathbb{R}$	0.5
	d)	Differentiability of $G$ at 0.....0.25	0.5
		Calculus of $G'(0)$ .....0.25	

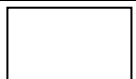
الصفحة 1 5	<p>الامتحان الوطني الموحد للبكالوريا المسالك الدولية – خيار إنجليزية الدورة العادية 2018 -الموضوع-</p>	<p>المملكة المغربية وزارة التربية الوطنية والتكوين المهني والتعليم العالي والبحث العلمي</p> <p>المركز الوطني للتقويم والامتحانات والتوجيه</p>
★★	NS 24E	

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- The duration of the exam is 4 hours.
- The exam contains 5 independent exercises.
- The exercises can be treated in the order chosen by the candidate.

- Exercise 1 concerns the algebraic structures.....(3.5pts)
- Exercise 2 concerns the arithmetic .....(3 pts)
- Exercise 3 concerns the complex numbers .....(3.5pts)
- Exercise 4 concerns the analysis.....(7.5pts)
- Exercise 5 concerns the analysis.....(2.5pts)

The use of calculators is not allowed  
The use of red pens on answers sheets is not permitted



**EXERCISE 1:** (3.5 points)

We recall that  $(\mathbb{C}, +, \times)$  is a commutative field and that  $(M_2(\mathbb{C}), +, \cdot)$  is a real vector space

and that  $(M_2(\mathbb{C}), +, ')$  is a commutative ring with unity  $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ , its additive identity is

the null matrix  $O = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

For every  $(x, y) \in \mathbb{C}^2$  we put  $M(x, y) = \begin{pmatrix} x & -2y \\ y & x+2y \end{pmatrix}$

and we consider the set  $E = \{M(x, y) / (x, y) \in \mathbb{C}^2\}$

0.25 1- Show that  $E$  is a sub-group of the group  $(M_2(\mathbb{C}), +)$

0.25 2- a) Show that  $E$  is a sub-space of the real vector space  $(M_2(\mathbb{C}), +, \cdot)$

0.5 b) We put  $J = M(0, 1)$ . Show that  $(I, J)$  is a basis of the real vector space  $(E, +, \cdot)$

0.5 3-a) Show that  $E$  is stable in  $(M_2(\mathbb{C}), ')$

0.5 b) Show that  $(E, +, ')$  is a commutative ring.

4- Let  $j$  be the mapping from  $\mathbb{C}^*$  to  $M_2(\mathbb{C})$  defined by :

$$(\forall (x, y) \in \mathbb{C}^2 - \{(0, 0)\}) ; \varphi(x + iy) = M(x + y, -y) = \begin{pmatrix} x + y & 2y \\ -y & x - y \end{pmatrix}$$

0.5 a) Show that  $j$  is an homomorphism from  $(\mathbb{C}^*, ')$  to  $(M_2(\mathbb{C}), ')$

0.5 b) We put  $E^* = E - \{O\}$ . Show that :  $j(\mathbb{C}^*) = E^*$

0.25 c) Deduce that  $(E^*, ')$  is a commutative group.

0.25 5- Show that  $(E, +, ')$  is a commutative field .

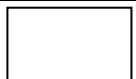
**EXERCISE 2:** (3 points)

Let  $p$  be a prime number such that  $p = 3 + 4k$  ( $k \in \mathbb{N}^*$ )

0.5 1- Show that for all integer number  $x$ , if  $x^2 \equiv 1 [p]$  then  $x^{p-5} \equiv 1 [p]$

2- Let  $x$  be an integer number verifying :  $x^{p-5} \equiv 1 [p]$

0.5 a) Show that  $x$  and  $p$  are coprime.



- 0.5 b) Show that :  $x^{p-1} \equiv 1 \pmod{p}$
- 0.5 c) Verify that :  $2 + (k-1)(p-1) = k(p-5)$
- 0.5 d) Deduce that :  $x^2 \equiv 1 \pmod{p}$
- 0.5 3) Solve in  $\mathbb{Z}$  the equation :  $x^{62} \equiv 1 \pmod{67}$

**EXERCISE 3:** (3.5 points)

Let  $m$  be a complex number.

I- We consider in the set of the complex numbers  $\mathbb{C}$  the equation  $(E_m)$  with variable  $z$  :

$$z^2 + (im + 2)z + im + 2 - m = 0$$

- 0.25 1-a) Verify that  $\Delta = (im - 2i)^2$  is the discriminant of the equation  $(E_m)$
- 0.5 b) Give according the values of  $m$  , the set of the solutions of the equation  $(E_m)$
- 0.5 2- For  $m = i\sqrt{2}$  , rewrite the two solutions of the equation  $(E_m)$  in exponential polar form.

II- The complex plane is attached to a direct orthonormal coordinate system  $(O; \vec{u}, \vec{v})$

We consider the points  $A$  ,  $\Omega$  ,  $M$  and  $M'$  , with respective affixes  $a = -1 - i$  ,  $\omega = i$  ,  $m$  and  $m' = -im - 1 + i$

- 1- Let  $R$  be the rotation with angle  $-\frac{\pi}{2}$  which send  $M$  to  $M'$
- 0.25 a) Verify that  $\Omega$  is the center of the rotation  $R$
- 0.5 b) Determine the affix  $b$  of  $B$  , where  $B$  is the point verifying :  $A = R(B)$
- 0.5 2-a) Verify that :  $m' - a = \frac{\omega - a}{\omega - b}(m - b)$
- 0.5 b) Deduce that the points  $A$  ,  $M$  and  $M'$  are collinear if and only if the points  $A$  ,  $B$  ,  $\Omega$  and  $M$  are cocyclic.
- 0.5 c) Show that the set of the points  $M$  such that the points  $A$  ,  $M$  and  $M'$  are collinear is a circle which center and radius are to be determined.

**EXERCISE 4:** (7.5 points)

**PART I**

- 0.5 1-a) Show that :  $(\forall x \in ]0, +\infty[)$  ;  $\int_0^x \frac{t}{1+t} dt = x - \ln(1+x)$
- b) By using the change of variable  $u = t^2$  , show that :
- 0.5  $(\forall x \in ]0, +\infty[)$ ;  $\int_0^x \frac{t}{1+t} dt = \frac{1}{2} \int_0^{x^2} \frac{1}{1+\sqrt{u}} du$

0.5 c) Deduce that :  $(\forall x \in ]0, +\infty[)$  ;  $\frac{1}{2(1+x)} \leq \frac{x - \ln(1+x)}{x^2} \leq \frac{1}{2}$

0.25 2- Determine :  $\lim_{x \rightarrow 0^+} \frac{x - \ln(1+x)}{x^2}$

**PART II**

We consider the numerical function  $f$  defined on  $]0, +\infty[$  by :

$$\begin{cases} f(x) = \left(\frac{x+1}{x}\right) \ln(1+x) ; x \neq 0 \\ f(0) = 1 \end{cases}$$

and let  $(C)$  be the representative curve of  $f$  in an orthonormal coordinate system  $(O; \vec{i}, \vec{j})$

0.25 1-a) Show that  $f$  is right-continuous at the point 0

b) Show that  $f$  is differentiable on the right at the point 0

0.5 (One can use the result of the question I-2)

0.75 c) Compute :  $\lim_{x \rightarrow +\infty} f(x)$  ,  $\lim_{x \rightarrow +\infty} \frac{f(x)}{x}$  and interpret graphically the obtained result.

0.5 2- a) Show that  $f$  is differentiable on  $]0, +\infty[$  , and verify that :

$$(\forall x \in ]0, +\infty[) ; f'(x) = \frac{x - \ln(1+x)}{x^2}$$

0.25 b) Deduce that  $f$  is strictly increasing on  $]0, +\infty[$

0.25 c) Verify that :  $f(]0, +\infty[) = ]1, +\infty[$

0.5 3- Plot the curve  $(C)$  (the right half-tangent at the point 0 must be plotted)

**PART III**

1- Let  $g$  be the numerical function defined on  $]0, +\infty[$  by:  $g(x) = f(x) - x$

0.5 a) Show that :  $(\forall x \in ]0, +\infty[) ; 0 \leq f'(x) \leq \frac{1}{2}$

0.5 b) Deduce that  $g$  is strictly decreasing on  $]0, +\infty[$  and show that  $g(]0, +\infty[) = ]-\infty, 1[$

0.25 c) Show that the equation  $f(x) = x$  admits a unique solution  $\alpha$  in  $]0, +\infty[$

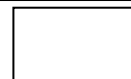
2- Let  $a$  be a real number in  $]0, +\infty[$  .

Consider the sequence  $(u_n)_{n \geq 0}$  defined by :  $u_0 = a$  and  $(\forall n \in \mathbb{N}) ; u_{n+1} = f(u_n)$

0.25 a) Demonstrate that :  $(\forall n \in \mathbb{N}) ; u_n > 0$

0.5 b) Show that :  $(\forall n \in \mathbb{N}) ; |u_{n+1} - \alpha| \leq \frac{1}{2} |u_n - \alpha|$





0.5

c) Prove by induction that :  $(\forall n \in \mathbb{N}) ; |u_n - \alpha| \leq \left(\frac{1}{2}\right)^n |a - \alpha|$

0.25

d) Show that the sequence  $(u_n)_{n \geq 0}$  converge to  $\alpha$

**EXERCISE 5 :** (2.5 points)

Consider the numerical function  $F$  defined on  $\mathbb{R}$  by:  $F(x) = \int_0^x e^{t^2} dt$

0.5

1- Show that  $F$  is continuous and is strictly increasing on  $\mathbb{R}$

0.5

2-a) Show that :  $(\forall x \in ]0, +\infty[) ; F(x) \geq x$  , and then deduce  $\lim_{x \rightarrow +\infty} F(x)$

0.5

b) Show that  $F$  is odd , and then deduce  $\lim_{x \rightarrow -\infty} F(x)$

0.5

c) Show that  $F$  is a bijection from  $\mathbb{R}$  to  $\mathbb{R}$

0.5

d) Show that the inverse bijection  $G$  of the function  $F$  is differentiable at 0 and compute  $G'(0)$

END