المملكة المغربية
وزارة التربية الوطنية والتكوين المهني والتعليم العالي والبحث العلمي

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المركز الوطني للتقويم والامتحانات والتوجيه

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المسالك الدولية – خيار إنجليزية
الدورة العادية 2019
- الموضوع -

NS24E

المادة	الرياضيات	مدة الانجاز	4
الشعبة أو المسلك	شعبة العلوم الرياضية : (أ) و (ب) – خيار أنجليزية	المعامل	9

- The duration of the exam is 4 hours.
- The exam contains 4 independent exercises.
- The exercises can be treated in the order chosen by the candidate.

- Exercise 1 concerns the algebraic structures ....(3.5 pts)
- Exercise 2 concerns the complex numbers ....(3.5 pts)
- Exercise 3 concerns the arithmetic ....(3 pts)
- Exercise 4 concerns the analysis ....(10 pts)

The use of calculators is not allowed
The use of red pens on answers sheets is not permitted

EXERCISE1 : (3.5 points)

Recall that $(\mathbb{C},+,\times)$ is a commutative field and that $(M_2(\mathbb{R}),+,\times)$ is a ring with unity $I=\begin{pmatrix}1&0\\0&1\end{pmatrix}$; its additive identity is the null matrix $O=\begin{pmatrix}0&0\\0&0\end{pmatrix}$

Let $*$ be the law internal composition defined on $\mathbb{C}$ by :

$$(\forall(x,y)\in\mathbb{R}^2)\ (\forall(a,b)\in\mathbb{R}^2)\quad ;\quad (x+yi)*(a+bi)=xa+(x^2b+a^2y)i$$

0.25 1-a) Show that $*$ is commutative on $\mathbb{C}$

0.5 b) Show that $*$ is associative on $\mathbb{C}$

0.25 c) Show that $*$ posses an identity element e which must be determined.

0.25 d) Let $(x,y)\in\mathbb{R}^*\times\mathbb{R}$; show that the complex number $x+yi$ admits the complex number $\frac{1}{x}-\frac{y}{x^4}i$ as inverse for the law internal composition $*$

2- Consider the sub-set E of $\mathbb{C}$ defined by : $E=\{x+yi\,/\,x\in\mathbb{R}_+^*\ ;\ y\in\mathbb{R}\}$

0.25 a) Show that E is stable in $(\mathbb{C},*)$

0.5 b) Show that $(E,*)$ is a commutative group.

3- Consider the set $G=\{1+yi\,/\,y\in\mathbb{R}\}$

0.5 Show that G is a sub-group of the group $(E,*)$

4-Consider the set $F=\left\{M(x,y)=\begin{pmatrix}x&y\\0&x\end{pmatrix}/\,x\in\mathbb{R}_+^*\ ;\ y\in\mathbb{R}\right\}$

0.25 a) Show that F is stable in $(M_2(\mathbb{R}),\times)$

0.5 b) Let φ be the mapping from E to F which associates to every complex number $x+yi$ of E the matrix $M(x^2,y)=\begin{pmatrix}x^2&y\\0&x^2\end{pmatrix}$ in F

Show that φ is an isomorphism from $(E,*)$ to $(F,\times)$

0.25 c) Deduce that $(F,\times)$ is a commutative group.

EXERCISE 2 :(3.5 points)

Let m be a complex number which is **not a real number** ($m\in\mathbb{C}-\mathbb{R}$)

I-Consider in $\mathbb{C}$ the equation, with the variable z, defined by :

$$(E)\ :\ z^2-(1+i)(1+m)z+2im=0$$

0.25 1-a) Show that the discriminant of the equation (E) is a non-zero complex number.

0.5 b) Determine z_1 and z_2 , the two solutions of the equation (E)

2- Assume in this question that : $m = e^{i\theta}$ where $0 < \theta < \pi$

0.5 a) Determine the module and an argument of $z_1 + z_2$

0.25 b) Show that if $z_1 z_2 \in \mathbb{R}$ then $z_1 + z_2 = 2i$

II- The complex plane is attached to a direct orthonormal coordinate system $(O;\vec{u},\vec{v})$.

Consider the following points :

A with affix $a = 1+i$, B with affix $b = (1+i)m$, C with affix $c = 1-i$, D the image of the point B by the rotation with center O and angle $\frac{\pi}{2}$ and Ω the midpoint of $[CD]$.

0.5 1- a) Show that the affix of the point Ω is $\omega = \frac{(1-i)(1-m)}{2}$

0.25 b) Calculate $\frac{b-a}{\omega}$

0.5 c) Deduce that $(O\Omega) \perp (AB)$ and that $AB = 2O\Omega$

2-The line $(O\Omega)$ intersects the line (AB) at the point H with affix h

0.5 a) Show that $\frac{h-a}{b-a}$ is real and that $\frac{h}{b-a}$ is pur imaginary.

0.25 b) Deduce h in term of m

EXERCISE3 : (3 points)

Admit that 2969 (The current amazigh's year) is prime number.

Let n and m be two natural numbers verifying : $n^8 + m^8 \equiv 0 \; [2969]$

1- Assume in this question that 2969 **do not divides** n

0.5 a) By using the BEZOUT's theorem, show that: $(\exists u \in \mathbb{Z})$; $u \times n \equiv 1 \; [2969]$

0.5 b) Deduce that : $(u \times m)^8 \equiv -1 \; [2969]$ and that : $(u \times m)^{2968} \equiv -1 \; [2969]$

(One can use : $2968 = 8 \times 371$)

0.5 c) Show that 2969 do not divides $u \times m$

0.5 d) Deduce that we also have : $(u \times m)^{2968} \equiv 1 \; [2969]$

0.5 2-a) By using the last results show that 2969 **divides** n

0.5 b) Show that : $n^8 + m^8 \equiv 0 \, [2969] \Leftrightarrow n \equiv 0 [2969] \; and \; m \equiv 0 [2969]$

EXERCISE4 : (10 points)

PART I : Consider the function f defined on $\mathbb{R}$ by : $f(x)=4x\left(e^{-x}+\frac{1}{2}x-1\right)$

and let (C) be its representative curve in an orthonormal coordinate system $(O;\vec{i},\vec{j})$

0.5 1- Calculate $\lim\limits_{x\to-\infty} f(x)$ and $\lim\limits_{x\to+\infty} f(x)$

0.5 2- a) Show that f is differentiable on $\mathbb{R}$, and that $(\forall x\in\mathbb{R})$; $f'(x)=4\left(e^{-x}-1\right)(1-x)$

0.75 b) Study the variations of f on $\mathbb{R}$, and set its table of variations.

0.5 c) Show that there exists a unique real α in the interval $\left]\frac{3}{2},2\right[$ such that $f(\alpha)=0$

(We give $e^{\frac{3}{2}}=4,5$)

0.25 d) Verify that : $e^{-\alpha}=1-\frac{\alpha}{2}$

0.5 3-a) By appling the ROLLE's theorem to the function f', show that there exists a real number x_0 in the interval $]0,1[$ such that : $f''(x_0)=0$

0.5 b) By appling the mean value theorem to the function f'', show that, for every x different from x_0 in the interval $[0,1]$, we have : $\frac{f''(x)}{x-x_0}>0$

0.25 c) Deduce that $I(x_0,f(x_0))$ is an inflecion point of the curve (C) .

0.5 4-a) Study the infinite branchs of the curve (C)

0.5 b) Plot the curve (C) in the coordinate system $(O;\vec{i},\vec{j})$.

(We take : $\|\vec{i}\|=\|\vec{j}\|=1cm$ and $f(1)=-0.5$ and it is not required to plot the point I)

0.25 5-a) Verify that : $(\forall x\in]-\infty,\alpha])$; $f(x)\leq 0$

0.75 b) Show that : $\int_0^\alpha f(x)dx=\frac{2}{3}\alpha\left(\alpha^2-3\right)$, and deduce that : $\frac{3}{2}<\alpha\leq\sqrt{3}$

0.5 c) Calculate in term of α , in cm^2 , the area of the region of the plane enclosed by the curve (C) and the lines with respective equations $y=0$, $x=0$ and $x=\alpha$

PART II : Consider the numerical sequence $(u_n)_{n\in\mathbb{N}}$ defined by :

$$u_0<\alpha \quad \text{and} \quad (\forall n\in\mathbb{N}) \quad ; \quad u_{n+1}=f(u_n)+u_n$$

0.5 1-a) Show by induction that : $(\forall n\in\mathbb{N})$; $u_n<\alpha$ (use the question 5-a) of PART I)

0.25 b) Deduce that the sequence $(u_n)_{n\in\mathbb{N}}$ is decreasing.

2- **Suppose that** $0\leq u_0$ and put $(\forall x\in\mathbb{R})$; $g(x)=e^{-x}+\frac{1}{2}x-\frac{3}{4}$

0.5 a) Show that: $(\forall x\in\mathbb{R})$; $g(x)>0$ (we take : $\ln 2=0.69$)

0.5 b) By using the result of the last question, show that : $(\forall n \in \mathbb{N}) \quad ; \quad 0 \leq u_n$

(Notice that : $f(x)+x=4xg(x)$)

0.25 c) Show that the sequence $(u_n)_{n\in\mathbb{N}}$ is convergent.

0.5 d) Compute $\lim\limits_{n\to+\infty} u_n$

3- **Suppose that** $u_0 < 0$

0.5 a) Show that : $(\forall n \in \mathbb{N}) \quad ; \quad u_{n+1}-u_n \leq f(u_0)$

0.5 b) Show that : $(\forall n \in \mathbb{N}) \quad ; \quad u_n \leq u_0 + nf(u_0)$

0.25 c) Deduce : $\lim\limits_{n\to+\infty} u_n$

END