الامتحان الوطني الموحد للبكالوريا

المسالك الدولية

الدورة العادية 2020

- عناصر الإجابة –

المملكة المغربية
وزارة التربية الوطنية والتكوين المهني والتعليم العالي والبحث العلمي
المركز الوطني للتقويم والامتحانات

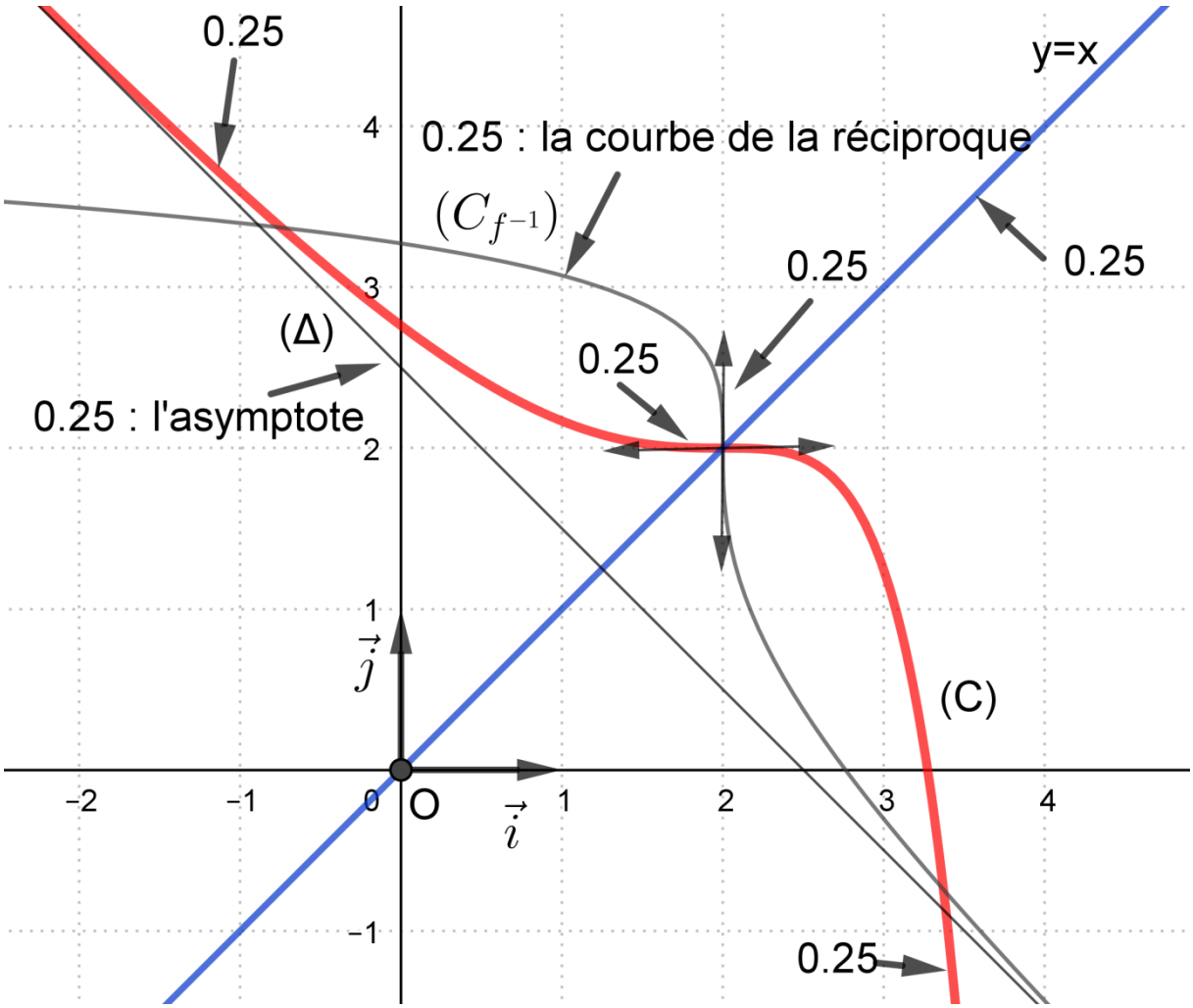
SSSSSSSSSSSSSSSSSS | NR 22E

المادة	الرياضيات	مدة الإنجاز	3
الشعبة أو المسلك	شعبة العلوم التجريبية مسلك علوم الحياة والأرض ومسلك العلوم الفيزيائية (خيار إنجليزية)	المعامل	7

On prendra en compte les différentes étapes de la solution et on acceptera toute méthode correcte .

	Numéros des questions	Notes	Eléments de réponses
Exercice 1	1	0.25	
	2	0.5	
	3-a	1	0.5 pour le premier encadrement et 0.5 pour le deuxième
	3-b	0.5	
	4-a	0.75	
	4-b	1	0.5 pour $v_n = \left(\frac{2}{5}\right)^n$ et 0.5 pour u_n en fonction de n
Exercice 2	1-a	0.5	
	1-b	1	0.5 pour chaque solution
	2-a	0.75	0.5 pour la vérification et 0.25 pour la déduction .
	2-b	0.5	0.25 pour chaque forme trigonométrique
	2-c	0.5	
	3-a	0.5	
	3-b	0.25	
	3-c	0.25	O est isocèle de sommet OBC Le triangle
	3-d	0.75	0.5 pour l'égalité et 0.25 pour la déduction .
Exercice 3	1-a	0.5	
	1-b	0.5	
	1-c	0.5	
	1-d	0.5	0.5 pour l'encadrement et 0.5 pour la limite
	2-a	0.75	
	2-b	0.75	
Problème	1	0.5	0.25 pour chaque limite
	2-a	0.5	On accepte toute méthode correcte
	2-b	0.75	0.25 pour l'équation et 0.25 pour la position relative dans chaque intervalle .
	3	0.5	0.25 pour la limite et 0.25 pour l'interprétation géométrique
	4-a	0.5	
	4-b	0.25	La mention de $f'(2)$ dans le tableau de variation n'est pas nécessaire

Problème	5	0.75	0.25 pour le calcul de la dérivée seconde et 0.5 pour le point d'inflexion
	6	0.5	
	7	1	Voir le graphe ci-dessous
	8-a	0.5	
	8-b	0.75	Voir le graphe ci-dessous
	8-c	0.5	



الامتحان الوطني الموحد للبكالوريا
المسالك الدولية
الدورة العادية 2020
- الموضوع –

SSSSSSSSSSSSSSSSSS | NS 22E

المملكة المغربية
وزارة التربية الوطنية والتكوين المهني والتعليم العالي والبحث العلمي
المركز الوطني للتقويم والامتحانات

3	مدة الإنجاز	الرياضيات	المادة
7	المعامل	شعبة العلوم التجريبية مسلك علوم الحياة والأرض ومسلك العلوم الفيزيائية (خيار إنجليزية)	الشعبة أو المسلك

GENERAL INSTRUCTIONS

- ✓ **The use of non- programmable calculator is allowed ;**
- ✓ **The exercises can be treated in the preferred order by the candidate ;**
- ✓ **The use of red color when writing solutions is to be avoided.**

COMPONENTS OF THE EXAM

The exam consists of three exercises and a problem , independent of each other according to the fields as follows :

Exercise 1	**numerical sequences**	**4 points**
Exercise 2	**Complex numbers**	**5 points**
Exercise 3	**Limits, differentiability and calculating integrals**	**4 points**
Problem	**Study of numerical function**	**7 points**

- ✓ $\overline{z}$ **denotes the conjugate of the complex number** z
- ✓ **ln denotes the Napierian logarithm function**

Exercise 1 : (4 points)

Let (u_n) be the numerical sequence defined by $u_0=\frac{3}{2}$ and $u_{n+1}=\frac{2u_n}{2u_n+5}$ for every natural number n

0.25 **1) calculate u_1**

0.5 **2) Show by induction that $u_n>0$ for every natural number n**

3) a) Show that: $0<u_{n+1}\le\frac{2}{5}u_n$; for every natural number n

1 **then deduce that , $0<u_n\le\frac{3}{2}\left(\frac{2}{5}\right)^n$ for every natural number n .**

0.5 **b) Calculate $\lim u_n$**

4) we consider the numerical sequence (v_n) defined by $v_n=\frac{4u_n}{2u_n+3}$ for every natural number n

0.75 **a) Show that (v_n) is a geometrical sequence of reason $\frac{2}{5}$**

1 **b) Determine v_n in terms of n therefore deduce u_n in terms of n for every natural number n.**

Exercise 2 : (5 points)

1) In the set of complex numbers $\mathbb{C}$ we consider the equation $(E):z^2-2\left(\sqrt{2}+\sqrt{6}\right)z+16=0$

0.5 **a) Verify that the discriminant of the equation (E) is $\Delta=-4\left(\sqrt{6}-\sqrt{2}\right)^2$**

1 **b) Deduce the solutions of equation (E) .**

2) Let the complex numbers $a=\left(\sqrt{6}+\sqrt{2}\right)+i\left(\sqrt{6}-\sqrt{2}\right)$, $b=1+i\sqrt{3}$ and $c=\sqrt{2}+i\sqrt{2}$

0.75 **a) Verify that $b\overline{c}=a$ and deduce that $ac=4b$**

0.5 **b) Write the complex numbers b and c in trigonometric form.**

0.5 **c) Deduce that $a=4\left(\cos\frac{\pi}{12}+i\sin\frac{\pi}{12}\right)$**

3) In the complex plane referred to an orthonormal direct coordinate system $(O,\vec{u};\vec{v})$,
we consider the points B, C and D of respective affixes b, c and d such that $d=a^4$
Let z be the affix of a point M in the complex plane and z' the affix of the point M' image of M by the rotation R with center O and angle $\frac{\pi}{12}$

0.5 **a) Verify that $z'=\frac{1}{4}az$**

0.25 **b) Determine the image of the point C by the rotation R**

0.25 **c) Determine the nature of the triangle OBC .**

0.75 **d) Show that $a^4=128b$ and deduce that the points O,B and D are collinear.**

Exercise 3 : (4 points)

Consider the numerical function g defined on $]0;+\infty[$ by $g(x)=2\sqrt{x}-2-\ln x$

0.5 **1) a) Show that for every x in $]0;+\infty[$; $g'(x)=\dfrac{\sqrt{x}-1}{x}$**

0.5 **b) Show that g is increasing on $[1;+\infty[$;**

0.5 **c) Deduce that for every x in $[1;+\infty[$; $0\le \ln x\le 2\sqrt{x}$; (Notice that $2\sqrt{x}-2\le 2\sqrt{x}$)**

1 **d) Show that for every x in $[1;+\infty[$: $0\le \dfrac{(\ln x)^3}{x^2}\le \dfrac{8}{\sqrt{x}}$ therefore deduce $\lim\limits_{x\to+\infty}\dfrac{(\ln x)^3}{x^2}$;**

0.75 **2) a) Show that the function G defined by $G(x)=x\left(-1+\dfrac{4}{3}\sqrt{x}-\ln x\right)$ is a primitive of the function g on $]0;+\infty[$.**

0.75 **b) Calculate the integral $\int_1^4 g(x)dx$.**

Problem : (7 points)

Consider the numerical function f defined on $\mathbb{R}$ by $f(x)=-x+\dfrac{5}{2}-\dfrac{1}{2}e^{x-2}\left(e^{x-2}-4\right)$ and (C) the curve of f in an orthonormal coordinate system $\left(O,\vec{i};\vec{j}\right)$ (unit: $2cm$)

0.5 **1) Show that $\lim\limits_{x\to-\infty}f(x)=+\infty$ and $\lim\limits_{x\to+\infty}f(x)=-\infty$**

0.5 **2) a) Show that the line (Δ) of equation $y=-x+\dfrac{5}{2}$ is an asymptote to the curve (C) near $-\infty$.**

0.75 **b) Solve the equation $e^{x-2}-4=0$, therefore show that the curve (C) is above (Δ) on the interval $]-\infty,2+\ln 4]$ and below (Δ) on the interval $[2+\ln 4,+\infty[$**

0.5 **3) Show that $\lim\limits_{x\to+\infty}\dfrac{f(x)}{x}=-\infty$ and interpret geometrically the obtained result**

0.5 **4) a) Show that for every x in $\mathbb{R}$: $f'(x)=-\left(e^{x-2}-1\right)^2$**

0.25 **b) Set up the table of variations of the function f**

0.75 **5) Calculate $f''(x)$ for every x in $\mathbb{R}$ therefore show that $A(2,2)$ is an inflection point of the curve (C)**

0.5 **6) Show that the equation $f(x)=0$ admits an unique solution α such that $2+\ln 3<\alpha<2+\ln 4$**

1 **7) Sketch the line (Δ) and the curve (C) in the same coordinate system $\left(O,\vec{i};\vec{j}\right)$**

(Take $\ln 2\approx 0,7$ and $\ln 3\approx 1,1$)

0.5	**8)a) Show that the function f admits an inverse function f^{-1} defined on $\square$.**
0.75	**b) Sketch in the same coordinate system $\left(O,\vec{i};\vec{j}\right)$ the curve of the function f^{-1} (Notice that the line (Δ) is perpendicular to the first bisector of coordinate system)**
0.5	**c) Calculate $\left(f^{-1}\right)'(2-\ln 3)$ (Notice that $f^{-1}(2-\ln 3)=2+\ln 3$)**