الامتحان الوطني الموحد للبكالوريا
المسالك الدولية
الدورة العادية 2020
- الموضوع -

المملكة المغربية
وزارة التربية الوطنية والتكوين المهني والتعليم العالي والبحث العلمي
المركز الوطني للتقويم والامتحانات

SSSSSSSSSSSSSSSSSSSS | NS 24E

المادة	الرياضيات	مدة الإنجاز	4
الشعبة أو المسلك	شعبة العلوم الرياضية (أ) و (ب) (خيار إنجليزية)	المعامل	9

- The duration of the exam is 4 hours.
- The exam is made up of (5) pages numbered from 1/5 to 5/5
- The exam includes four independent exercises.
- **The candidate must treat the EXERCISE3 and EXERCISE4, and choose to treat**
  EXERCISE1 or EXERCISE2
- **The candidate must perform totaly three (3) exercises:**

EXERCISE1 which concerns arithmetic **(optional)**..**3.5 points**

- or

EXERCISE2 which concerns algebraic structures **(optional)**...**3.5 points**

- EXERCISE3 which concerns complex numbers **(obligatory)** ....**3.5 points**

- EXERCISE4 which concerns analysis **(obligatory)** ....**13 points**

The use of the calculator is not allowed

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choose to treat EXERCISE1 or EXERCISE2
and treat obligatory EXERCISE3 and EXERCISE4

<u>EXERCISE1</u> :(3.5 points/optional)

(If you choose to treat EXERCISE1, you do not treat EXERCISE2)

Consider in ¢ ´ ¢ the equation $(D)\ :\ 7x^3 - 13y = 5$

1- Let $(x,y)\in \mathbb{Z}\times\mathbb{Z}$ be a solution of the equation (D)

0.5 a) Show that x and 13 are coprime.

0.5 b) Deduce that : $x^{12}\equiv 1\ [13]$

1 c) Show that : $x^{3}\equiv 10\ [13]$

0.5 d) Deduce that : $x^{12}\equiv 3\ [13]$

1 2- Deduce from the previous questions, that the equation (D) does not admit a solution in $\mathbb{Z}\times\mathbb{Z}$

EXERCISE2 : (3.5 points/optional)

(If you choose to treat EXERCISE2, you do not treat EXERCISE1)

$M_2(\mathbb{R})$ denotes the set of square matrices of order two.

We recall that $(M_2(\mathbb{R}),+,\times)$ is a non commutative ring with unit $I=\begin{pmatrix}1&0\\0&1\end{pmatrix}$ and that $(\mathbb{R}^*,\times)$ is a commutative group.

Consider the sub-set E of $M_2(\mathbb{R})$ defined by : $E=\left\{\begin{pmatrix}1&x\\0&y\end{pmatrix}/\ x\in\mathbb{R}\ \ and\ y\in\mathbb{R}^*\right\}$.

0.5 1- a) Show that E is a stable part of $(M_2(\mathbb{R}),\times)$

0.5 b) Show that the multiplication is not commutative in E

0.5 c) Verify that : $(\forall x\in\mathbb{R})(\forall y\in\mathbb{R}^*)$; $\begin{pmatrix}1&x\\0&y\end{pmatrix}\times\begin{pmatrix}1&-\frac{x}{y}\\0&\frac{1}{y}\end{pmatrix}=\begin{pmatrix}1&-\frac{x}{y}\\0&\frac{1}{y}\end{pmatrix}\times\begin{pmatrix}1&x\\0&y\end{pmatrix}=\begin{pmatrix}1&0\\0&1\end{pmatrix}$

0.5 2-Show that $(E,\times)$ is a non commutative group.

3- Consider the sub-set F of E defined by : $F=\left\{M(x)=\begin{pmatrix}1&x-1\\0&x\end{pmatrix}/\ x\in\mathbb{R}^*\right\}$

0.5 a) Show that the map φ defined by $(\forall x\in\mathbb{R}^*)$; $\varphi(x)=M(x)$ is an homomorphism from $(\mathbb{R}^*,\times)$ to $(E,\times)$.

1 b) Deduce that $(F,\times)$ is a commutative group whose neutral element will be specified.

EXERCISE3 :(3.5 points/obligatory)

Let m be a non-zero complex number.

First part :

Consider in $\mathbb{C}$ the equation with variable z, (E) : $z^3-2mz^2+2m^2z-m^3=0$

0.5 1- Solve in $\mathbb{C}$ the equation (E) (notice that m is a solution of the equation (E))

2- z_1 and z_2 denote the other two solutions of the equation (E), which are different from m

0.25 a) Verify that: $\frac{1}{z_1}+\frac{1}{z_2}=\frac{1}{m}$

0.5 b) In case where $m=1+e^{i\frac{\pi}{3}}$, write in algebrac form z_1 and z_2

Second part :

The complex plane is attached to a direct orthonormal coordinate system $(O;\vec{u},\vec{v})$

Consider the points A and B with respective affixes $a=me^{i\frac{\pi}{3}}$ and $b=me^{-i\frac{\pi}{3}}$

We denote P the center of the rotation with angle $\left(\frac{\pi}{2}\right)$ which sends O to A

and Q the center of the rotation with angle $\left(\frac{\pi}{2}\right)$ which sends A to B

and R the center of the rotation with angle $\left(\frac{\pi}{2}\right)$ which sends B to O

0.25 1- Show that the points O, A and B are non collinear.

1 2-a) Show that the affix of P is $p=m\frac{\sqrt{2}}{2}e^{i\frac{7\pi}{12}}$ and that the affix of R is $r=m\frac{\sqrt{2}}{2}e^{-i\frac{7\pi}{12}}$

0.5 b) Show that the affix of Q is $q=m\sqrt{2}\sin\left(\frac{7\pi}{12}\right)$

0.5 3-Show that $OQ=PR$ and that the lines (OQ) and (PR) are perpendicular.

<u>EXERCISE4</u> :(13 points/obligatory)

First part :

Consider the numerical function f defined on the interval $I=[0;+\infty[$ by :

$$f(0)=0 \quad \text{and} \quad \forall x\in\,]0;+\infty[,\quad f(x)=x^3\ln\left(1+\frac{1}{x}\right)$$

Let (C) be its curve in an orthonormal coordinate system $(O;\vec{i},\vec{j})$

(We will take $\|\vec{i}\|=\|\vec{j}\|=1cm$)

0.5 1- By using the mean values theorem to the function $t\mapsto \ln(t)$ on the interval $[x;x+1]$,

show that : (P) $(\forall x\in\,]0;+\infty[)$; $\frac{1}{x+1}<\ln\left(1+\frac{1}{x}\right)<\frac{1}{x}$

0.5 2- a) By using the proposition (P) show that the function is differentiable on the right at 0

0.5 b) By using the proposition (P) show that the curve (C) admits a parabolic branch whose direction must be specified.

0.75 3-a) Show that the fonction f is differentiable on $]0;+\infty[$ and :

$$(\forall x\in\,]0;+\infty[)\quad ;\quad f'(x)=3x^2\left(\ln\left(1+\frac{1}{x}\right)-\frac{1}{3(1+x)}\right)$$

0.5 b) Deduce that the function f is strictly increasing on I (one can use the propostion (P))

0.25 c) Set the table of variations of f

4- For $x\in\,]0;+\infty[$,we put : $g(x)=\dfrac{f(x)}{x}$

0.75 a) Verify that : $(\forall x\in\,]0;+\infty[)\ ;\ g'(x)=2x\left(\ln\left(1+\frac{1}{x}\right)-\frac{1}{2(1+x)}\right)$, and then deduce that the function g is strictly increasing on $\mathbb{R}_+^*$

0.5 b) Show that the equation $g(x)=1$ admits in $\mathbb{R}_+^*$, a unique solution denoted α and verify that $\alpha\in\,]1;2[$ (We will take $\ln 2=0.7$ and $\ln\frac{3}{2}=1.5$)

0.5 c) Deduce that the only solutions of the equation $f(x)=x$ are 0 and α

0.5 5-a) Plot the curve (C).

(We will specify the right half-tangent at O and the parabolic branch of (C))

0.25 b) Show that f is a bijection from I to I (f^{-1} denotes its inverse bijection)

Seconde part :

We consider the numerical sequence $(u_n)_{n\geq 0}$ defined by :

$$0<u_0<\alpha \quad\text{and}\quad (\forall n\in\mathbb{N})\ ;\ u_{n+1}=f^{-1}(u_n)$$

0.5 1-Show by induction that : $(\forall n\in\mathbb{N})\ ;\ 0<u_n<\alpha$

0.5 2-a) Show that $g(]0;\alpha[)=]0;1[$

0.5 b) Deduce that the sequence $(u_n)_{n\geq 0}$ is strictly increasing.

0.25 c) Show that the sequence $(u_n)_{n\geq 0}$ is convergent.

0.5 3- Determine $\lim\limits_{n\to+\infty} u_n$

Third part :

We consider the function F defined on the interval I by : $(\forall x \in I)$; $F(x)=\int_x^1 f(t)\,dt$

0.5 1-a) Study according to the values of x, the sign of $f(x)$

0.5 b) Show that the function F is differentiable on I and determine its differentiable Function F'

0.25 c) Deduce that F is strictly decreasing on I

0.5 2-a) Show that : $(\forall x \in [1;+\infty[)$; $F(x) \leq (1-x)\ln 2$

0.25 b) Deduce $\lim\limits_{x \to +\infty} F(x)$

0.5 3-a) By using an integration by parts show that :

$$\forall x \in]0;+\infty[\quad ; \quad F(x)=\frac{\ln 2}{4}-\frac{x^4}{4}\ln\left(1+\frac{1}{x}\right)+\frac{1}{4}\int_x^1 \frac{t^3}{t+1}\,dt$$

0.5 b) Calculate $\int_x^1 \frac{t^3}{t+1}\,dt$ for all $x \in]0;+\infty[$ (notice that: $\frac{t^3}{1+t}=t^2-t+1-\frac{1}{1+t}$)

0.5 c) Deduce that : $\forall x \in]0;+\infty[\quad ; \quad F(x)=\frac{5}{24}-\frac{x^3}{12}+\frac{x^2}{8}-\frac{x}{4}+\frac{1}{4}\ln(1+x)-\frac{x^4}{4}\ln\left(1+\frac{1}{x}\right)$

0.5 d) Calculate $\lim\limits_{x \to 0^+} F(x)$, and then deduce the value of $\int_0^1 f(t)\,dt$

4-For any non-zero natural number n , we put: $v_n=\sum_{k=0}^{k=n-1}\left(F\left(\frac{2k+1}{2n}\right)-F\left(\frac{k}{n}\right)\right)$

0.5 a) Show that for all $n \in \mathbb{N}^*$ and for all $k \in \{0,1,\ldots\ldots,n-1\}$:

$$-\frac{1}{2n} f\left(\frac{2k+1}{2n}\right) \leq F\left(\frac{2k+1}{2n}\right)-F\left(\frac{k}{n}\right) \leq -\frac{1}{2n} f\left(\frac{k}{n}\right)$$

0.5 b) Deduce that: $(\forall n \in \mathbb{N}^*)$; $-\frac{1}{2n}\sum_{k=1}^{k=n} f\left(\frac{k}{n}\right) \leq v_n \leq -\frac{1}{2n}\sum_{k=0}^{k=n-1} f\left(\frac{k}{n}\right)$

(notice that : $\frac{2k+1}{2n}<\frac{k+1}{n}$)

0.25 c) Show that the numerical sequence $(v_n)_{n \in \mathbb{N}^*}$ is convergent and determine its limit.

END

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الامتحان الوطني الموحد للبكالوريا

المسالك الدولية

الدورة العادية 2020

- عناصر الإجابة -

SSSSSSSSSSSSSSSSSSSS	NR 24E

المادة	الرياضيات	مدة الإنجاز	4
الشعبة أو المسلك	شعبة العلوم الرياضية (أ) و (ب) (خيار إنجليزية)	المعامل	9

N.B : If the candidate treats the two optional exercises (totally or partially) , we give the best score of the two obtained score (and not the sum of these two scores).

EXERCISE1		Elements of answers	M
1-	a)	If d is a positive common divisor to x and 13 then it is a common divisor to 13 and 5 so $d = 1$	0.5
	b)	13 is a prime number and 13 and x are coprime , and we use the FERMAT's theorem	0.5
	c)	We have : $7x^3 \equiv 5\ [13]$ and then $x^3 \equiv 2\times 5\ [13]$ because : $2\times 7 \equiv 1\ [13]$	1
	d)	We have $x^3 \equiv 10\ [13]$ so $(x^3)^4 \equiv 10^4\ [13]$ and then $x^{12} \equiv 3\ [13]$	0.5
2-		If $(x,y) \in \mathbb{Z}\times\mathbb{Z}$ is a solution of (E) then by the question 1- We have $x^{12} \equiv 1\ [13]$ and $x^{12} \equiv 3\ [13]$ so $3 \equiv 1\ [13]$ which is false	1

EXERCISE2		Elements of answers	M
1-	a)	Stability of E in $M_2(\mathbb{R}, \times)$	0.5
	b)	The non commutativity of the multiplication in E	0.5
	c)	Verification	0.5
2-		$(E,\times)$ is a no commutative group	0.5
3-	a)	φ is an homomorphism	0.5
	b)	φ is a morphism and $\varphi(\mathbb{R}^*) = F$ and $(\mathbb{R}^*,\times)$ is a commutative group……………………………………………………………….0.5 Its neutral element is $\varphi(1) = I$ …………………………………….0.5	1

EXERCISE3		Eléments of answers	M
First part			
1-		$(E) \Leftrightarrow (z-m)\left(z^2-mz+m^2\right)=0$ The solutions of (E) are : m and $\frac{1+i\sqrt{3}}{2}m=e^{i\frac{\pi}{3}}m$ and $\frac{1-i\sqrt{3}}{2}m=e^{-i\frac{\pi}{3}}m$	0.5
2-	a)	We verify that $\frac{1}{z_1}+\frac{1}{z_2}=\frac{z_1+z_2}{z_1z_2}=\frac{m}{m^2}$	0.25
	b)	We find : $z_1=i\sqrt{3}$ and $z_2=\sqrt{3}\left(\frac{\sqrt{3}}{2}-i\frac{1}{2}\right)$	0.5
Second part :			
1-		The points O , A and B are not collinear	0.25
2-	a)	Calculus of p ....0.5 Calculus of r 0.5	1
	b)	Calculus of q	0.5
3-		We have $\frac{p-r}{q}=i$ and we deduce that : $OQ=PR$ ....0.25 and $(OQ)\perp(PR)$ 0.25	0.5

EXERCISE4		Elements of answers	M
First part :			
1-		$\forall x>0\ \exists c_x \in]x;x+1[\ ;\quad ln(x+1)-ln\,x=\frac{1}{c_x}$ ....0.25 The inequalitys : $\frac{1}{x+1}<ln\left(1+\frac{1}{x}\right)<\frac{1}{x}$ 0.25	0.5
2-	a)	We have : $\frac{x^2}{1+x}<\frac{f(x)}{x}<x$ then $\lim_{x\to 0^+}\frac{f(x)}{x}=0$ and then f is differentiable on right at 0	0.5

	b)	We have : $\frac{x^2}{1+x} < \frac{f(x)}{x}$ then $\lim\limits_{x\to+\infty} \frac{f(x)}{x} = +\infty$ ....0.25 (C) admits a P.B with the y-axis as direction ..0.25	0.5
3-	a)	f differntiable on $]0,+\infty[$ ....0.25 Calculus of $f'(x)$ 0.5	0.75
	b)	We have : $ln\left(1+\frac{1}{x}\right) \frac{1}{3(1+x)} > ln\left(1+\frac{1}{x}\right) \frac{1}{1+x} > 0$ then $f'(x) > 0$ so f is strictly increasing...	0.5
	c)	Variation table of f	0.25
4-	a)	Calculus of $g'(x)$...0.5 We have : $ln\left(1+\frac{1}{x}\right) \frac{1}{2(1+x)} > ln\left(1+\frac{1}{x}\right) \frac{1}{1+x} > 0$ then $g'(x) > 0$ f is strictly increasing 0.25	0.75
	b)	g is a bijection from $]0;+\infty[$ to $]0;+\infty[$ and $1 \in]0;+\infty[$ We use I.V.T for the existence of α and the strict monocity for the unicity of .... 0.25 We verify that $g(1) < 1 < g(2)$...0.25	0.5
	c)	The solutions of the equation: $f(x) = x \Leftrightarrow x = 0$ or $g(x) = 1$	0.5
5-	a)	The curve (C)	0.5
	b)	f bijection	0.25

Second part:

1-	Induction and monocity (increasing) of f^{-1} and the fact that $f^{-1}(\alpha) = \alpha$ and $f^{-1}(0) = 0$	0.5

2-	a)	$g(]0;\alpha[) =]0;1[$	0.5
	b)	For $0 < x < \alpha$, we have $0 < g(x) < 1$ Since $0 < u_n < \alpha$,then $0 < f(u_n) < u_n$ so $0 < u_n < f^{-1}(u_n) = u_{n+1}$	0.5
	c)	Strictly increasing and upper bounded sequence	0.25
3-		If we put : $l = \lim\limits_{n \to +\infty} u_n$ then we have $0 < u_0 \leq l \leq \alpha$ because $(\forall n \geq 1) \; ; \; 0 < u_0 < u_n < \alpha$ And since f^{-1} is continous on $[0;\alpha]$ (In particular at l) then l is a solution of the equation $f(x) = x$ and so $l = \alpha$	0.5
Third part :			
1-	a)	f is positive so if $0 \leq x \leq 1$ we have $F(x) \geq 0$ and if $x \geq 1$ we have $F(x) \leq 0$	0.5
	b)	F is differentiable on I because f is continuous on I ....0.25 And $(\forall x \in I) \; ; \; F'(x) = -f(x)$..............................0.25	0.5
	c)	$(\forall x \in I) \; ; \; F'(x) = -f(x) \leq 0$ and $F'(x) = 0 \Leftrightarrow x = 0$	0.25
2-	a)	We have $(\forall x \geq 1) \; ; \; f(x) \geq ln(2)$ so $(\forall x > 1) \; ; \; \int_1^x f(t)dt \geq (x-1)ln2$	0.5
	b)	$\lim\limits_{x \to +\infty} F(x) = -\infty$	0.25
3-	a)	Integration by part	0.5
	b)	$\int_x^1 \frac{t^3}{t+1}dt = \frac{5}{6} - ln2 - \frac{x^3}{3} + \frac{x^2}{2} - x + ln(1+x)$	0.5
	c)	Calculus of $F(x)$	0.5

	d)	We have $\lim\limits_{x\to 0^+} F(x)=\frac{5}{24}$ ....0.25 F is continuous on right at 0 (since it is continuous on I), then $\int_0^1 f(t)dt = F(0)=\lim\limits_{x\to 0^+} F(x)=\frac{5}{24}$ 0.25	0.5
4-	a)	- Apply the theorem or the inequality of finite increments to the function F on the interval $\left[\frac{k}{n},\frac{2k+1}{2n}\right]$ with $\left(\forall x\in\left[\frac{k}{n},\frac{2k+1}{2n}\right]\right)$; $f\left(\frac{k}{n}\right)\le f(x)\le f\left(\frac{2k+1}{2n}\right)$	0.5
	b)	We notice that : $\frac{2k+1}{2n}<\frac{k+1}{n}$	0.5
	c)	$\frac{1}{n}\sum\limits_{k=0}^{k=n-1} f\left(\frac{k}{n}\right)$ and $\frac{1}{n}\sum\limits_{k=0}^{k=n-1} f\left(\frac{k+1}{n}\right)=\frac{1}{n}\sum\limits_{k=1}^{k=n} f\left(\frac{k}{n}\right)$ are the Riemann sums associated to the continuous function f on the segment $[0,1]$ therefore the two sequences $\left(\frac{1}{n}\sum\limits_{k=0}^{k=n-1} f\left(\frac{k}{n}\right)\right)_n$ and $\left(\frac{1}{n}\sum\limits_{k=1}^{k=n} f\left(\frac{k}{n}\right)\right)_n$ are convergent and have the same limit which is $F(0)=\int_0^1 f(x)dx=\frac{5}{24}$ so the sequence (v_n) is convergent and has $-\frac{1}{2}F(0)=-\frac{5}{48}$ as limit	0.25