المملكة المغربية
وزارة التربية الوطنية
والتكوين المهني
والتعليم العالي والبحث العلمي
المركز الوطني للتقويم والامتحانات

# الامتحان الوطني الموحد للبكالوريا
## المسالك الدولية
الدورة العادية 2021
- عناصر الإجابة -

SSSSSSSSSSSSSSSSSSSS | NR 24E

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|---|---|---|---|
| المادة | الرياضيات | مدة الإنجاز | 4h |
| الشعبة أو المسلك | شعبة العلوم الرياضية (أ) و (ب) (خيار إنجليزية) | المعامل | 9 |

| Exercise 1 | | | Elements of solutions | Marks |
|---|---|---|---|---|
| Part I | 1- | a) | - We show that : $\lim\limits_{x\to+\infty}\left(f_n(x)-nx+2\right)=0$ ------------<br>- The line with the equation $y=nx-2$ is an asymptote to $(C_n)$ at $+\infty$.------------------------------- | 0.25<br>0.25 |
| | | b) | We show that the line $(\Delta_n): y=nx$ is an asymptote to $(C_n)$ at $-\infty$<br>**Mark 0 is assigned to incomplete answer** | 0.5 |
| | 2- | a) | - Show that $f_n$ is differentiable on $\mathbb{R}$ ..................<br>- Justify the expression of $f_n'(x)$ ........................... | 0.25<br>0.25 |
| | | b) | Show the inequality | 0.5 |
| | | c) | $f_0$ is strictly decreasing on $\mathbb{R}$<br>And $(\forall n\geq 1)$ , $f_n$ is strictly increasing on $\mathbb{R}$<br>**Marks 0 is assigned to incomplete answer** | 0.5 |
| | 3- | a) | Determination of the quation of the tangent. | 0.5 |
| | | b) | The proof that $I(0;-1)$ is the unique inflexion point of $(C_n)$ | 0.5 |
| | 4- | | Graphical representation of $(C_0)$ ....................<br>Graphical representation of $(C_2)$ .................... | 0.25<br>0.25 |
| | 5- | a) | Calculate the integrale. | 0.5 |
| | | b) | Justify the calculation of $\lim\limits_{t\to+\infty} A(t)$ | 0.5 |
| Part II | 1- | a) | Justify the existence and the uniqueness of $\alpha$<br>**Mark 0 is assigned to incomplete answer** | 0.5 |
| | | b) | Show the inequality | 0.5 |
| | 2- | a) | Show the inequality | 0.5 |
| | | b) | Show the inequality | 0.5 |
| | | c) | Show the convergence of the sequence $\alpha$ | 0.5 |
| Part III | 1- | a) | Justify the existence and the uniqueness of $x_n$ | 0.5 |

| | | | | |
|---|---|---|---|---|
| | | b) | Justify the two inequalities | 0.5 |
| | 2- | a) | Show the inequality | 0.5 |
| | | b) | Show that the sequence is strictly decreasing | 0.5 |
| | | c) | Justify that the sequence $(x_n)_{n\geq 2}$ is convergente | 0.5 |
| | 3- | a) | Justify the two inqualities | 0.5 |
| | | b) | Calculate $\lim\limits_{n\to+\infty} x_n$<br>Justify that : $\lim\limits_{n\to+\infty} nx_n = 1$ | 0.25<br>0.25 |
| | 4- | a) | Show the inequality | 0.5 |
| | | b) | The deduction of the limite | 0.5 |

| Exercise 2 | | Elements of solutions | Marks |
|---|---|---|---|
| 1- | a) | Solving the quation<br>**All solutions are accepted** | 0.5 |
| | b) | The write of the two solutions in exponential form. | 0.25x2 |
| 2- | a) | Proof of the two inequalities | 0.5x2 |
| | b) | Calculating the ratio | 0.5 |
| | c) | Determination of the nature of the triangle $PQD$ | 0.5 |
| 3- | a) | Proof of the inequality | 0.5 |
| | b) | Proof of the cocyclic points. | 0.5 |

| Exercise 3 | | | Elements of solutions | Marks |
|---|---|---|---|---|
| **Part I** | 1- | | Verification | 0.25 |
| | 2- | | Solving the equation $(E)$ with justification of all steps<br>**Mark 0 is assigned to incomplete answer** | 0.75 |
| **Part II** | 1- | a) | -Justify that b $x$ and $43$ are coprimesont ......<br>- Justify the congruence............... | 0.25<br>0.25 |
| | | b) | - Justify the congruence ............................<br>- Justify the congruence ............................. | 0.25<br>0.25 |
| | 2- | | Give the set of the solutions of the equation $(F)$ | 0.5 |
| **Part III** | 1- | a) | -Justify that $x \equiv 11\ [43]$ ............................<br>- Justify that $x \equiv 10\ [47]$ .............................. | 0.25<br>0.25 |
| | | b) | Justify the congruence ............................<br>**Mark 0 is assigned to incomplete answer** | 0.5 |
| | 2- | | Give the set of the solutions of the system $(S)$<br>**Mark 0 is assinged to incomplete answer** | 0.5 |

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| | | | |
|---|---|---|---|
| 4h | مدة الإنجاز | الرياضيات | المادة |
| 9 | المعامل | شعبة العلوم الرياضية (أ) و (ب) (خيار إنجليزية) | الشعبة أو المسلك |

**- The duration of the exam is four hours**

**-The exam consists of 3 independent exercises**

**- The exercises can be treated in the order chosen by the candidate**

**- Exercise1 concerns analysis ……….……….…(12 pts)**

**- Exercise2 concerns complex numbers………….(4 pts)**

**- Exercise3 concerns arithmetic ………….……...(4 pts)**

**The use of the calculator is not allowed**

**The use of red color is not permitted**

**EXERCISE1** : (12 points)

For every natural number $n$ we consider the function $f_n$ defined on $\mathbb{R}$ by :

$$f_n(x)=\frac{-2e^x}{1+e^x}+nx$$

And let $(C_n)$ be its representetive curve in an orthonormal coordinate system $(O,\vec{i},\vec{j})$.

(We will take $\|\vec{i}\|=\|\vec{j}\|=1cm$)

**Part I** :

0.5 1-a) Calculate $\lim\limits_{x\to+\infty}\left(f_n(x)-nx+2\right)$ , then interpret graphically the obtained result.

0.5 b) Show that the curve $(C_n)$ admits, at $-\infty$ , an asymptote $(\Delta_n)$ for which you will determine a cartesian equation .

0.5 2-a) Show that the function $f_n$ is differentiable on $\mathbb{R}$ and that :

$$(\forall x\in\mathbb{R}) \ ; \ f_n'(x)=\frac{-2e^x}{(1+e^x)^2}+n$$

0.5 b) Show that : $(\forall x\in\mathbb{R}) \ ; \ \dfrac{4e^x}{(1+e^x)^2}\leq 1$

0.5 c) Deduce the sense of variation of the function $f_n$ on $\mathbb{R}$

(We distinguish the two cases : $n=0$ and $n\geq 1$ )

0.5 3-a) Determine the equation of the tangent to $(C_n)$ at the point $I$ with abscissa $0$

0.5 b) Show that the point $I$ is the unique inflection point of the curve $(C_n)$

0.5 4- Plot in the same coordinate system, the curves $(C_0)$ and $(C_2)$.

5- For all real number $t>0$ , $A(t)$ denotes the area of the region delimited by the curve $(C_n)$ and the lines with equations : $y=nx-2$ , $x=0$ and $x=t$

0.5 a) Calculate $A(t)$ for every $t>0$

0.5 b) Calculate $\lim\limits_{t\to+\infty}A(t)$

**Part II :**

Consider the numerical sequence $(u_n)_{n\geq 0}$ defined by :

$$u_0=0 \quad \text{and} \quad (\forall n\in\mathbb{N}) \ ; \ u_{n+1}=f_0(u_n)$$

0.5 1-a) Show that the equation $f_0(x)=x$ admits a unique solution $\alpha$ in $\mathbb{R}$

0.5 b) Show that : $(\forall x\in\mathbb{R}) \ ; \ |f_0'(x)|\leq\dfrac{1}{2}$

0.5 2-a) Show that : $(\forall n\in\mathbb{N}) \ ; \ |u_{n+1}-\alpha|\leq\dfrac{1}{2}|u_n-\alpha|$

0.5 b) Deduce that : $(\forall n \in \mathbb{N})$ ; $|u_n - \alpha| \leq \left(\frac{1}{2}\right)^n |\alpha|$

0.5 c) Show that the sequence $(u_n)_{n\geq 0}$ converges to $\alpha$ .

**Part III** :

Suppose in this part that $n \geq 2$

0.5 1- a) Show that for all natural number $n \geq 2$ , there exists a unique real number $x_n$ which is a solution of the equation $f_n(x) = 0$

0.5 b) Show that for all natural number $n \geq 2$ , $0 < x_n < 1$

( we will take $\frac{2e}{1+e} < 1.47$ )

0.5 2-a) Show that for all natural number $n \geq 2$, $f_{n+1}(x_n) > 0$

0.5 b) Deduce that the sequence $(x_n)_{n\geq 2}$ is strictly decreasing.

0.5 c) Show that the sequence $(x_n)_{n\geq 2}$ is convergent.

0.5 3-a) Sow that for all natural number $n \geq 2$, $\frac{1}{n} < x_n < \frac{1}{n}\left(\frac{2e}{1+e}\right)$

0.5 b) Deduce $\lim\limits_{n\to+\infty} x_n$ then show that : $\lim\limits_{n\to+\infty} nx_n = 1$

0.5 4-a) Show that for all natural number $n \geq 2$, we have $x_n \leq x_2$

0.5 b) Deduce $\lim\limits_{n\to+\infty} (x_n)^n$

**<u>EXERCISE2</u>** : (4 points)

Let $a$ , $b$ and $c$ be three complex numbers differing zero such that : $a+b \neq c$

0.5 1-a) Solve in $\mathbb{C}$ the equation with variable $z$ $(E)$ : $z^2 - (a+b+c)z + c(a+b) = 0$

0.5 b) We suppose in this question that : $a = i$ , $b = e^{i\frac{\pi}{3}}$ and $c = a - b$

Write the solutions of the equation $(E)$ in exponential form .

2- The complex plane is attached to a direct orthonormal coordinate system $(O,\vec{u},\vec{v})$ .

Consider the three points $A(a)$ , $B(b)$ and $C(c)$ assumed non-collinear.

Let $P(p)$ be the center of the rotation with angle $\frac{\pi}{2}$ which sends $B$ to $A$

And $Q(q)$ be the center of the rotation with angle $\left(-\frac{\pi}{2}\right)$ which sends $C$ to $A$

And $D(d)$ be the midpoint of the segment $[BC]$

1 a) Show that : $2p = b + a + (a-b)i$ and $2q = c + a + (c-a)i$

0.5 b) Calculate : $\dfrac{p-d}{q-d}$

0.5 c) Deduce the nature of the triangle $PDQ$

3- Let $E$ be the symmetric point of $B$ with respect to $P$, and $F$ be the symmetric point of $C$ with respect to $Q$ and $K$ be the midpoint of the segment $[EF]$ .

0.5 a) Show that the affix of $K$ is $k = a + \dfrac{i}{2}(c-b)$

0.5 b) Show that the points $K$ , $P$ , $Q$ and $D$ are cocyclic.

**EXERCISE3** : (4 points)

**Part I** : Consider in $\mathbb{Z}\times\mathbb{Z}$ the equation $(E)$ : $47x - 43y = 1$

0.25 1- Verify that the couple $(11,12)$ is a particular solution of the equation $(E)$

0.75 2- Solve in $\mathbb{Z}\times\mathbb{Z}$ the equation $(E)$

**Part II** : Consider in $\mathbb{Z}$ the equation $(F)$ : $x^{41} \equiv 4 \ [43]$

1- Let $x$ be a solution of the equation $(F)$

0.5 a) Show that $x$ and $43$ are coprime, then deduce that : $x^{42} \equiv 1 \ [43]$

0.5 b) Show that : $4x \equiv 1 \ [43]$, then deduce that : $x \equiv 11 \ [43]$

0.5 2- Give the set of the solutions in $\mathbb{Z}$ of the equation $(F)$

**Part III** : Consider in $\mathbb{Z}$ the system with two equations $(S) : \begin{cases} x^{41} \equiv 4 \ [43] \\ x^{47} \equiv 10 \ [47] \end{cases}$

1- Let $x$ be a solution of the system $(S)$

0.5 a) Show that $x$ is a solution of the system $(S') : \begin{cases} x \equiv 11 \ [43] \\ x \equiv 10 \ [47] \end{cases}$

0.5 b) Deduce that : $x \equiv 527 \ [2021]$ (One can use Part I)

0.5 2- Give the set of the solutions in $\mathbb{Z}$ of the system $(S)$

**END**